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Local times and related sample Path properties of certain self-similar processes

By

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1. Introduction

Let $X(t)$, $t \in R_+$, be a real-valued measurable stochastic process. We say that X is H-self-similar (abbrev. H-ss), $H > 0$, if for any $c > 0$ $\{X(ct)\}$ and $\{c^H X(t)\}$ have the same finite dimensional distributions. We say that X is of stationary increments (abbrev. si) if for any $b \geq 0$ $\{X(t+b) - X(b)\}$ and $X(t) - X(0)\}$ have the same finite dimensional distributions. We say that X is symmetric α-stable (abbrev. $S\alpha S$) if all finite linear combinations $\sum_{i=1}^n a_i X(t_i)$ are symmetric α-stable random variables. A bibliographical guide to the development of self-similar processes can be found in Taqqu [18]. We also mention that there are two important classes of H-ss si $S\alpha S$ processes, namely linear fractional stable and (real) harmonizable fractional stable processes, which are defined respectively by

$$\Delta_{H,\alpha}(a, b; t) = \int_{-\infty}^{\infty} \{a[(t-u)_+^{H-1/\alpha} - (-u)_+^{H-1/\alpha}] + b[(t-u)_-^{H-1/\alpha} - (-u)_-^{H-1/\alpha}]\} Z_\alpha(\mathrm{d}u), \quad \text{and}$$

$$\Psi_{H,\alpha}(a, b; t) = Re \int_{-\infty}^{\infty} \frac{e^{itu} - 1}{iu} (au_+^{1-H-1/\alpha} + bu_-^{1-H-1/\alpha}) \tilde{Z}_\alpha(\mathrm{d}u),$$

where $0 < H < 1$, $0 < \alpha < 2$, $H \neq 1/\alpha$, a and $b \in R$ such that $a^2 + b^2 > 0$, and Z_α and $\tilde{Z}_\alpha$ are respectively real and complex symmetric Lévy α-stable motions. See Cambanis–Maejima [6] for detailed discussions on the distributional properties and the limiting theorems of these two processes.

The investigation on the "fine" sample path properties of ss processes has been stimulated a lot by the intensive works of Vervaat [19, 20]. Regarded as a contribution to this expanding topic, it is the purpose of this paper to study the local times and the related path properties of certain ss processes. In this aspect, we mention that Kôno [10] and Kôno-Maejima [11] proved the existence of square-integrable local times and Nolan [14, 15] discussed the joint continuity of local times for certain stable processes including $\Psi_{H,\alpha}(1, 1)$ [14, Proposition 4.9].

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