On the index of reducibility of parameter ideals
and Cohen-Macaulayness in a local ring

By

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1. Introduction

Let $A$ be a Noetherian local ring with maximal ideal $m$ and $a$ be an $m$-primary ideal of $A$. Then Northcott proved that the number of irreducible components of $a$, which is called the index of reducibility of $a$, is equal to the length of $\text{Hom}_A(A/m, A/a)$ [10]. In general, let $M$ be a finitely generated $A$-module and $N$ be a submodule of $M$ such that $M/N$ has finite length. Then the number of irreducible components of $N$ in $M$, which is also called the index of reducibility of $N$ in $M$, is equal to the length of $\text{Hom}_A(A/m, M/N)$. It is well-known that if $M$ is a Cohen-Macaulay $A$-module of dimension $d$, then the index of reducibility of a submodule $xM$ of $M$, where $x = x_1, \ldots, x_d$ is a system of parameters for $M$, depends only on $M$, and not on the choice of the system of parameters.

On the other hand, it is known that in a local ring $A$, if the index of reducibility of any parameter ideal is equal to one, then $A$ is Cohen-Macaulay, and hence Gorenstein [11]. But there are examples of a non-Cohen-Macaulay ring such that the index of reducibility of any parameter ideal is equal to a constant not depending on the choice of the system of parameters [3].

Concerning these results, the following question may be raised:

Let $A$ be a Noetherian local ring such that the index of reducibility of any parameter ideal for $A$ is equal to some constant. What makes $A$ Cohen-Macaulay?

The aim of this paper is to answer this question and to generalize it for modules.

2. Preliminaries

In this section, we state some definitions and recall some facts on a dualizing complex and on a module with finite local cohomologies. Throughout this paper, $A$ denotes a Noetherian local ring with maximal ideal $m$. Let

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