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A topological proof of Bott periodicity theorem and a characterization of BU

By

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1. Introduction

The purpose of this paper is to give a simple and topological proof of the Bott periodicity theorem $\mathbf{BU} \simeq \Omega^2\mathbf{BSU}$ and a characterization of $\mathbf{BU}$.

Let X be a finite type CW complex which is an H-space satisfying the following properties:

(1) as an algebra, $H^*(X;\mathbf{Z})=\mathbf{Z}[c_1, c_2, \cdots\cdots, c_n, \cdots]$ where $|c_i|=2i$;

(2) there exist two maps

$$j: \mathbf{CP}^\infty \to X \text{ and}$$

$$\lambda: S^2 \wedge X \to X$$

such that

$$(\lambda \circ (S^2 \wedge j))^*: H^*(X;\mathbf{Z}) \to H^*(S^2 \wedge \mathbf{CP}^\infty;\mathbf{Z}) \text{ is epic and}$$

$$Ad^2\lambda: X \to \Omega^2 X \qquad \text{is an H-map.}$$

Denote the homotopy fiber of $c_1: X \to \mathbf{CP}^\infty = K(\mathbf{Z}, 2)$ by $X\langle 2\rangle$. Then $Ad^2\lambda$ induces a map $Ad^2\lambda: X \to \Omega^2(X\langle 2\rangle)$ since $\Omega^2(X\langle 2\rangle)$ is the connected component of $\Omega^2 X$ containing the constant loop and X is connected. The map $\widetilde{Ad}^2\lambda$ is a homotopy equivalence (see Theorem 2.1).

Note that $\mathbf{BU}$ is an H-space and satisfies (1). Let λ be the classifying map of

$$\tilde{K}(\cdot) \to \tilde{K}(S^2 \wedge \cdot)$$

defined by

$$x \mapsto (\eta - 1) \hat{\otimes} x$$

and j be the classifying map of $\eta_\infty - 1$ where η (resp. η_∞) is the cannonical line

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