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The quasi-sure existence of solutions for differential equations on Wiener space

By

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1. Introduction

A.B. Cruzeiro [3] proved the almost-sure existence of solutions for differential equations on the Wiener space. In this paper, we extend the almost-sure existence to the *quasi-sure existence*, that is, the existence of solutions for all initial values except in a set of (r,p)-capacity 0 for all $r\geq 0$ and $p>1$. Here the capacity is associated with the Ornstein-Uhlenbeck operator. To show the quasi-sure existence we use the notion of Sobolev spaces of Banach valued functions (see Shigekawa [13] and Denis [4]). To be precise, let (X, H, μ) be an abstract Wiener space and A be a vector field on X which is a smooth mapping from X into H in the sense of Malliavin. Then under some conditions for A, which will be stated precisely in Theorem 5.5 below, the solutions $V_t(x)$ of the following differential equation exist for all $t \in \boldsymbol{R}$, quasi everywhere x(q.e. x).

$$(1.1)\qquad \begin{cases} (dV_t/dt)(x)=A(V_t(x)+x), \\ V_0(x)=0. \end{cases}$$

We first consider (1.1) in finite dimensional case. We show that for any $k\in N$, $(L^k V_t)$ exists for all $t\in\mathbf{R}$, μ-a.e. x and thereby (V_t) admits a quasi-continuous modification as a $C([0, T]\rightarrow X)$-valued function for any $T>0$. In finite dimensional case, one point has a positive (r, p)-capacity for sufficiently large r and p . Therefore we can show that a solution to (1.1) exists for every initial value $x\in X$. To deal with $(L V_t)$, for example, we have to consider the following differential equation:

$$(1.2)\qquad \frac{d}{dt} V_t(x)=B(V_t(x)+x),$$

$$(1.3)\qquad \frac{d}{dt}\nabla V_t(x)=\nabla B(V_t(x)+x)\cdot\nabla V_t(x)+\nabla B(V_t(x)+x),$$

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