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On the extremality for Teichmüller mappings

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1. Introduction

The universal Teichmüller space $T(1)$ is the totality of quasicircles with suitable normalization. This space $T(1)$ is universal in the sense that it contains the Teichmüller space of an arbitrary Riemann surface. There are several useful realization of $T(1)$. Among them the following Bers embedding method is now standard.

Consider $T(1)$ as the set of conformal mappings of the unit disk $E=\{|z|<1\}$ onto domains surrounded by quasicircles, and take the Schwarzian derivatives of them. Then we can show that $T(1)$, which is now considered as the set of those Schwarzian derivatives, is a bounded domain in the Banach space B_2 consisting of all φ holomorphic on E with the norm

$$\|\varphi\lambda^{-2}\|_\infty=\sup_{z\in E}|\varphi(z)\lambda(z)^{-2}|<\infty \quad , \tag{1.1}$$

where $\lambda^2(z)|dz|^2$ is the Poincaré metric on E. For the relationship between quasicircles and Schwarzian, we refer to the works of Nehari [21], Ahlfors and Weill [2], Gehring and Pommerenke [8], and the author's recent work [15]. More detail reference for Teichmüller space, one can consult with, for instance, Lehto [19], Gardiner [6], Nag [20], and Imayoshi and Taniguchi [18].

The above representation tells us that for describing the whole universal Teichmüller space, we need only those holomorphic functions on E with the order estimate

$$|\varphi(z)|=O\left(\left(\frac{1}{1-|z|}\right)^2\right) . \tag{1.2}$$

Because we know that the essential data for holomorphic quadratic differentials is the geometric structure of trajectories (or topologically, the foliation structure of them), we can assert that whole universal Teichmüller space can be controled by geometric structures of holomorphic functions φ on E satisfying the above order estimate.

This may lead us to the concept of the "Teichmüller model" of $T(1)$, which is the L^∞-theory originated from Teichmüller, and whose main tool is a

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