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Complex manifolds modeled on a complex Minkowski space

By

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§0. Introduction

In the present paper, we investigate the differential geometry of complex Finsler manifolds. The main purpose is to introduce a connection on a complex Finsler manifold as the transversal connection constructed by the same method as used in [3], and to discuss some properties of *complex manifolds modeled on a complex Minkowski space*, which is a complex version of the notion due to Ichijyō [8].

We denote by $\boldsymbol{C}^n$ the complex vector space of n-tuples of complex numbers. A function $f(\xi)$ defined on $\boldsymbol{C}^n$ is said to be a *Finsler metric* if it satisfies the following properties:

(i) $f(\xi)\geq 0$, the equality holds if and only if $\xi=(\xi^1, \dots, \xi^n)=\mathbf{0}$,

(ii) $f(\xi)$ is C^∞ on $\boldsymbol{C}^n - \{O\}$, and continuous on $\boldsymbol{C}^n$,

(iii) $f(\lambda\xi)=|\lambda|^2 f(\xi)$ for $\forall \lambda\in\boldsymbol{C}$,

(iv) $f(\xi)$ is strictly plurisubharmonic outside of the origin O, that is, the Hermitian matrix $(\partial^2 f/\partial\xi^\alpha\partial\bar{\xi}^\beta)$ is positive-definite.

The condition (iv) is equivalent to the strict pseudoconvexity of the indicatrix $I=\{\xi\in\boldsymbol{C}^n; f(\xi)<1\}$. Conversely, if a *complete proper circular domain* I in $\boldsymbol{C}^n$ with smooth boundary is strictly pseudoconvex, the Minkowski functional of I defines a Finsler metric on $\boldsymbol{C}^n$ whose indicatrix becomes the given I ([13]). Any Hermitian metric on $\boldsymbol{C}^n$ belongs to the class of Finsler metrics, and is characterized by one of the following three equivalent conditions (see Corollary 3.2 in [13]):

(1) The indicatrix I is biholomorphic to the unit ball in $\boldsymbol{C}^n$.

(2) The function $f(\xi)$ is C^∞ at the origin O.

(3) The function $f(\xi)$ is expressed as $f(\xi)=\sum_{i=1}^{n}\left|\sum_{m=1}^{n}A^i_m\xi^m\right|^2$ for $\exists\ (A^i_j)\in GL(n, \boldsymbol{C})$.

In the present paper, following to Ichijyō [9], we call a Finsler metric f on $\boldsymbol{C}^n$ a *complex Minkowski metric* on $\boldsymbol{C}^n$, and the pair $(\boldsymbol{C}^n, f)$ a *complex Minkowski space*.

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