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Morse function and attaching map

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1. Introduction

Let M be a closed manifold and f be a Morse function, a differentiable function on M with isolated, non-degenerate, critical points. Associated to a Morse function there is a cell complex L which is homotopy equivalent to M, see Milnor [M2]. In the following we identify M with such a cell complex L though there is not a canonical way to construct L from M. Therefore we assume that M has a cell structure. By $M^{(n)}$ we denote the n-skeleton of M, whereas we set $M^c=f^{-1}((-\infty, c])$ and $M_c=f^{-1}(c)$ for a real number c. Since there is a j-cell for each critical point of index j, we identify a cell with the corresponding critical point. Thus the integral homology group of M is computed as the homology group of the cell complex. For a j-cell α its bounday $\partial\alpha$ is described as follows: When we write $\partial\alpha=\Sigma n_i\cdot\beta_i$, then n_i is the intersection number of the unstable manifold of $-$*grad* f at α with the stable manifold of $-$*grad* f at β_i. Here we fix a Riemannian metric of M. That is, the composition of the attaching map of the j-cell α $M^{(j-1)}$ and the collapsing map

$$M^{(j-1)} \to M^{(j-1)}/M^{(j-2)} \approx \bigvee_i S^{j-1} \to S^{j-1}$$

(This composition shall be called the attaching map of the j-cell α to the $(j-1)$-cell β_i.) is described as the intersection number of the unstable and stable manifolds of critical points.

Now we assume that the Morse function f on the manifold M has no critical points of indices $j+1, \ldots, j+\ell-1$, where ℓ is a positive integer.

The unstable framed cobordism group consists of the equivalence clsses of n-dimensional manifolds embedded in S^{n+k} with a framing of the trivial normanl bundle and the group is isomorphic to $\pi_{n+k}(S^k)$, the homotopy group of maps from S^{n+k} to S^k, see Milnor [M1]. The purpose of this paper is to prove the following thorem and to give its application to the projective spaces.

Therrem. *The attaching map of the $j+\ell$-cell α to the j-cell β is described as the (transversal) intersection manifold of the unstable manifold of $-$grad f at α with the stable manifold of $-$grad f at β considered as a framed manifold embedded in the unstable manifold.*

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