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A coupling of infinite particle systems

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In this note we extend a coupling technique introduced in Mountford (1993) to a large class of interacting particle systems (IPSs) on the one dimensional lattice. A one dimensional IPS is a Markov process on state space D^Z where Z is the integers and (in this paper) D is some finite set of possible spin values. The generator for this process can be written as

$$\Omega f(\eta)=\sum_{T}\sum_{\nu\in D^T}(f(\nu\eta)-f(\eta))c_T(\eta,\nu)$$

where the first sum is over finite subsets of Z, T and where $\nu\eta$ denotes the configuration with $\nu\eta(y)$ equal to $\nu(y)$ if $y\in T$ and equal to $\eta(y)$ otherwise. The function $c_T(\nu,\eta)$ can be assumed to be zero if $\nu(y)=\eta(y)$ for some y in T. In this case for ν different from η on T, we should think of the process as satisfying

$$P[\eta_{t+dt}=\nu \text{ on T } |\eta_t]=c_T(\eta_t,\nu)dt+o(dt).$$

See Liggett (1985), especially section 1.3, for a discussion of existence questions. Throughout this paper we will assume that the process

is of *finite range* : there exists an $R<\infty$ so that $c_T(\,,\,)$ is zero if T has length greater than R and such that for any x in Z and T containing x of length at most R, $c_T(\nu,\eta)$ depends only on the spins $\eta(x-R)$, $\eta(x-R+1)$,,,,, $\eta(x)$,....$\eta(x+R)$.

and

has *bounded flip rates* : for each site x, $\sum_{x\in T}\sum_{\nu\in D^T}c_T(\nu,\eta)<1$. The bound of 1 is arbitrary, any bound can be reduced to 1 by rescaling time.

Given these hypotheses, there exists a unique Markov semigroup S(t). corresponding to operator Ω. It should be noted that if the "flip" functions c are translation invariant, then (perhaps after rescaling time) the bounded flip rates hypothesis is guaranteed once the finite range hypothesis is satisfied.

A probability measure v on D^Z is invariant for the process if for each f continuous on D^Z and for each t>0

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