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An example of regular (r, p)-capacity and essential self-adjointness of a diffusion operator in infinite dimensions

Dedicated to Professor Masatoshi Fukushima on his 60th birthday

By

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1. Introduction

The general theory of (r, p)-capacity has been developed by Fukushima-Kaneko [8] (see also [10]). In their theory, the regularity condition is fundamental.

To be precise, let X be a separable metric space and m be a finite Borel measure on X. Suppose that a symmetric Markovian semigroup $\{T_t\}$ on $L^2(X; m)$ is given. By the Markovian property, $\{T_t\}$ is a contraction semigroup on $L^p(X; m)$ for any $p \in [1, \infty)$. The Gamma transformation is defined by

$$V_r = \frac{1}{\Gamma(r/2)} \int_0^\infty t^{r/2-1} e^{-t} T_t dt.$$

Set $\mathscr{F}_{r,p} := V_r(L^p(X; m))$. Using $\mathscr{F}_{r,p}$, we can define the (r, p)-capacity $C_{r,p}$ as follows: for an open set G,

$$C_{r,p}(G) := \inf\{\|u\|_{r,p}^p;\ u \in \mathscr{F}_{r,p},\ u \geq 1\ m\text{-a.e. on } G\} \tag{1.1}$$

and for an arbitrary set $S \subseteq X$,

$$C_{r,p}(S) := \inf\{C_{r,p}(G);\ G \text{ is open and } G \supseteq S\}. \tag{1.2}$$

In this paper, we say that the (r, p)-capacity is *regular* if the following condition is satisfied:

(R) $\mathscr{F}_{r,p} \cap C_b(X)$ is dense in $\mathscr{F}_{r,p}$.

Here $C_b(X)$ denotes the set of all bounded continuous functions on X. Assuming the condition (R), Fukushima-Kaneko [8] proved the continuity from the below of the (r, p)-capacity.

The purpose of this paper is to give an example satisfying the condition (R). Let (B, H, μ) be an abstract Wiener space. We take a function $\rho \in W^{\infty, \infty-}$ with $\rho > 0$ μ-a.e. and fix it. We consider the following Dirichlet form in $L^2(\rho^2\mu)$:

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