J. Math. Kyoto Univ. (JMKYAZ)
35-4 (1995) 607-610

A remark on Brauer's height zero conjecture

By

Masafumi MURAI

Introduction

Let G be a finite group and p a prime. Let B be a p-block of G with defect group D. A well-known conjecture of R. Brauer asserts:

(HZ) Every irreducible character in B is of height 0 if and only if D is abelian.

This assertion is naturally divided into two parts:

(AHZ) If D is abelian, then every irreducible character in B is of height 0.

(HZA) If every irreducible character in B is of height 0, then D is abelian.

The assertion (AHZ) has been reduced to the case of quasi-simple groups by Berger and Knörr [2]. See also [10]. The assertion (HZA) has been shown to be true for p-solvable groups by Gluck and Wolf [6].

Let $k_0(B)$ be the number of irreducible characters of height 0 in B. Let $\tilde{B}$ be the Brauer correspondent of B in $N_G(D)$. Then the Alperin-McKay conjecture asserts:

(AM) $\quad k_0(B) = k_0(\tilde{B})$.

For p-solvable groups the assertion (AM) has been shown to be true, cf. Dade [3], Okuyama and Wajima [11].

On the other hand, R. Knörr and G. R. Robinson have shown that if the conjecture (AM) and Alperin's weight conjecture [1] are true, then so is the conjecture (AHZ), cf. [8, Proposition 5.6] for a more precise statement. Moreover E. C. Dade has given a conjecture which implies (AM) and Alperin's weight conjecture (and hence (AHZ)) are true, cf. [4, p. 188].

Here we prove the following

Theorem. *Assume that* (AM) *is true for the principal blocks of all finite groups and that* (HZA) *is true for the principal blocks of all simple groups. Then* (HZA) *is true for the principal blocks of all finite groups.*

Communicated by Prof. T. Hirai, October 27, 1994