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Abelian conformal field theory and $N = 2$ supercurves

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§. Introduction

Among models of conformal field theory, one of the simplest is the one with abelian gauge symmetry or what we call abelian conformal field theory. It has been studied by many people and from several view-points. We refer only a few of them [KNTY, ACKP, IMO, KSU 1-2], where one finds the notion of dressed moduli spaces and the so-called Krichever maps.

More recently, Ueno [U] studied it in parallel with conformal field theory with nonabelian gauge symmetry [TUY]. In particular, he adjoined the gauge condition by vertex operators among Fock spaces following a suggestion of Tsuchiya.

The aim of this paper is to give a geometric interpretation of this result by Ueno on abelian conformal field theory.

The basic strategy is as follows. First, the gauge condition on conformal blocks or correlators can be interpreted as the effect of localization à la Beilinson-Bernstein of representations of some infinite dimensional Lie algebra on certain "dressed" moduli space, cf. [B, BS].

Second, we want to consider the direct sum of infinitely many Fock spaces together with vertex operators. This amounts to consider the irreducible highest weight representations of Clifford algebra generated by free fermions corresponding to those vertex operators.

This suggests that we should consider geometric objects with fermionic degree of freedom. Thus we are led to consider an analog of Picard (or Jacobian) variety for algebraic supercurves with odd dimension $N = 2$ (abbreviated as $N = 2$ supercurves). We study here the space (Π-Picard variety) of locally free sheaves of rank $1|1$ with Π-symmetry (Π-invertible sheaves) introduced by Skornyakov [VMP, §4, M2, Ch. 2, §8].

Then one of the main results is the following:

Theorem (cf. 5.2.1). *The space of conformal blocks $\mathcal{V}(\mathfrak{X})$ equals a fiber of the localization of the given representation on the Π-Picard scheme of the supercurve $X^{(1)}$ of odd dimension $N = 2$ associated with an ordinary curve C.*

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