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Commutant algebra of Cartan-type Lie superalgebra $W(n)$

By

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Introduction

Undoubtedly, Weyl's classical reciprocity law is very important in Lie theory ([4]). It tells us that there is a correspondence between irreducible representations of general linear group $GL(V)$ in m-fold tensor space $V \otimes \cdots \otimes V$ and those of $\mathfrak{S}_m$ (symmetric group of degree m). The correspondence is one-to-one and so an irreducible representation of $\mathfrak{S}_m$ determines that of $GL(V)$ and vice versa. If we consider various m, then all the irreducible polynomial representations of $GL(V)$ appear in the decomposition, so one gets a classification of irreducible representations via those of $\mathfrak{S}_m$.

In the article [2], the first author studied an analogous phenomenon for a Cartan-type Lie algebra of vector fields. In the present paper, we want to do it for a Cartan-type Lie *superalgebra* $W(n)$. By definition $W(n)$ is a Lie superalgebra of all the superderivations on a Grassmann algebra $\Lambda(n)$ of n-variables (see [1]), so $W(n)$ acts on $\Lambda(n)$ naturally. We call it the natural representation of $W(n)$ and denote it by ψ. To study an analogue of Weyl's reciprocity, the first important thing is to calculate the commutant algebra of the natural representation of $W(n)$ in m-fold tensor Grassmann algebra $\otimes^m \Lambda(n)$. Let End $[m]$ be the set of all the maps from $[m]$ to $[m]$, where $[m] = \{1, 2, \cdots, m\}$, and denote the semigroup ring of End $[m]$ by $\mathfrak{E}_m$. There is the natural represntation of $\mathfrak{E}_m$ on $\otimes^m \Lambda(n)$ (see Section 1.3) and denote the image algebra of this representation by $\mathscr{E}_m$. Also we denote the commutant algebra of $\psi^{\otimes m}(W(n))$ in End $\otimes^m \Lambda(n)$ by $\mathscr{C}_m$ (see Section 1.2). One of the main results is the identification of the commutant algebra $\mathscr{C}_m$ and the semigroup ring $\mathscr{E}_m$ (Theorems 2.3 and 3.2). However, in Theorem 2.3 we assume $m \leq n$ (the rank n is larger then the power of tensor product m), and in Theorm 3.2 we restrict ourselves to the case $n = 1$. In the future studies, we want to clarify the relationship between $\mathscr{C}_m$ and $\mathscr{E}_m$ fo general m and n.

The another main result is Theorem 3.3, which says that the bicommutant algebra of $W(1)$ coincides with the image of its universal enveloping algebra $U(W(1))$. We consider it an analogy of Weyl's reciprocity for $W(1)$

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