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A probabilistic scheme for collapse of metrics

Dedicated to Professor Masatoshi Fukushima on his 60th birthday

By

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1. Introduction

After the development of the theory of collapse of Riemannian manifolds [1, 3], Ikeda and the first author spelled out the correspondence between the collapse of Riemannian metrics on a manifold and the convergence of the Brownian motions associated with them in [5, 8]. In [8], the first author employed the monotone convergence theorem for Dirichlet forms to investigate the convergence of resolvents, semigroups, and eigenvalues corresponding to the Laplace-Beltrami operators associated with the converging sequence of Riemannian metrics on a manifold. However, the advantage of the monotone convergence theorem bears much more than what was investigated in the paper. Indeed, we can establish a probabilistic scheme to treat the collapse of "metrics" on an infinite dimensional space such as a path group space over a Lie group, which is the main motivation of this paper.

From a point of view of the theory of Dirichlet forms, the based state space need not to be a manifold, and we can develop an analytic argument for generalized "Riemannian metrics" on a more general space. Namely, consider a separable metric space X as a "manifold" and a family of separable real Hilbert spaces H_x, $x \in X$ as a family of its tangent spaces at x. Then the space $\mathbf{S}$ of families A of non-negative definite symmetric operators $A(x): H_x \to H_x^*$ is regarded as a space of generalized "Riemannian metrics", where the symmetry and non-negativity are defined in a usual manner identifying H_x^* with H_x. Roughly speaking, our first aim is to see the convergence of associated bilinear forms, resolvents and semigroups when $A_n \in \mathbf{S}$ converges to A, and the second is to specify the limit bilinear form. For details, see Section 2.

A typical example covered by the above scheme is a path group

$$X \equiv \{\mathbf{x}: [0, 1] \to G : \mathbf{x} \text{ is continuous and } \mathbf{x}(0) = e\}$$

over a Lie group G with an AdG-invariant inner product $\langle \cdot , \cdot \rangle_{\mathscr{G}}$ on the Lie algebra $\mathscr{G}$. In this case, due to the group structure on X, all H_x coincide with a Hilbert space of functions $\mathbf{h}: [0, 1] \to \mathscr{G}$ with $\mathbf{h}(0) = 0$ which are abso-

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