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Physically reasonable solutions to steady compressible Navier-Stokes equations in 3D-exterior domains ($v_\infty = 0$)

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1. Introduction

In this paper we study the asymptotic properties of the kinetic and density fields of a compressible viscous Navier-Stokes fluid, filling a three dimensional domain exterior to a compact reagion Ω_c, when the prescribed velocity at infinity is zero. As far as the authors know, this is the first contribution in the subject. (The existence and uniqueness to this problem was studied in several papers of Matsumura and Nishida, Novotny and Padula, Novotny, Padula, see [14], [15], [19], [16], [21].)

The same problem, in the simpler case of incompressible Navier-Stokes fluids attracted mathematicians since the paper of Leray [12], who constructed (for the arbitrary size of external data) a solution of problem:

$$\Delta u + \nabla \Pi = -u\cdot\nabla u + f, \qquad \operatorname{div} u = 0\,, \qquad u|_{\partial\Omega_c} = 0 \tag{1.1}$$

(here u denotes the velocity and p the pressure), with the finite Dirichlet integral for the velocity (so called Leray solution). In 1965, Finn [4] proved (for small external forces) existence of solutions with the spatial decay of rate $|x|^{-1}$ for the velocity (so called physically reasonable solutions)[1]. He also proved, in [4], [5], [6] that any physically reasonable solution (if it exists) possesses the decay:

$$u \sim |x|^{-1},\ \nabla u \sim |x|^{-2}\lg|x|,\ \Pi \sim |x|^{-2}\lg|x|\,. \tag{1.2}$$

This statement was (for small data) in a certain sense improved by Borchers and Miyakawa [2]. They have proved existence of solutions in weak Lebesgue spaces $L^{3/2}_w(\Omega)$[2]. More precisely

$$u \sim |x|^{-1}\,, \qquad \nabla u \in L^{3/2}_w(\Omega)\,. \tag{1.3}$$

Similar result was derived independently by Galdi and Simader [10]:

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[1] Only recently, in 1991, Galdi [7] proved that the Leray solution was a physically reasonable one, provided external data are "small".

[2] Recall the definition of $L^{3/2}_w(\Omega)$, $1 < t < \infty$. It is a Bnach space of functions φ with the finite norm $\|\varphi\|_{t,w} = \sup_E\left[(\text{meas } E)^{-1+1/t}\int_E |\varphi|\,dx\right]$. (The supremum is taken over all bounded measurable subset of Ω.)