

On dynamics of hyperbolic rational semigroups

By

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1. Introduction

For a Riemann surface S , let $\text{End}(S)$ denote the set of all holomorphic endomorphisms of S . It is a semigroup with the semigroup operation being composition of functions. A *rational semigroup* is a subsemigroup of $\text{End}(\bar{\mathbf{C}})$ without any constant elements. Similarly, an *entire semigroup* is a subsemigroup of $\text{End}(\mathbf{C})$ without any constant elements. A rational semigroup G is called a *polynomial semigroup* if each $g \in G$ is a polynomial. When a rational or entire semigroup G is generated by $\{f_1, f_2, \dots, f_n, \dots\}$, we denote this situation by

$$G = \langle f_1, f_2, \dots, f_n, \dots \rangle,$$

The rational or entire semigroup generated by a single function g is denoted by $\langle g \rangle$. We denote the n -th iterate of f by f^n .

The study of rational semigroups is a generalization of the study of Kleinian groups, iteration of rational functions and systems of contraction maps related to self-similar sets in \mathbf{C} in fractal geometry. D. Sullivan pointed out that there are many points of similarity between Kleinian groups and iteration of rational functions in [Sul]. In view of the study of rational semigroups, we can show some basic results similar between Kleinian groups and iteration of rational functions. For example, limit sets of Kleinian groups, Julia sets of rational functions and self-similar sets in \mathbf{C} are *Julia sets* of rational semigroups. By Lemma 1.1.5.6, which is a result by A. Hinkkanen and G. J. Martin, the fixed points are dense in these sets. Several properties of dynamics of rational semigroups have been shown in [ZR], [GR], [HM1], [HM2], [S1] and [S2]. In 1992, the first study was investigated by W. Zhou and F. Ren ([ZR]). In 1996, the study of infinitely generated semigroup of meromorphic functions was investigated by Z. Gong and F. Ren ([GR]). In 1996, A. Hinkkanen and G. J. Martin studied about nearly abelian rational semigroups ([HM1]). They showed that Julia sets of finitely generated rational semigroups are uniformly perfect ([HM2]).

In this paper, we use the notations in [HM1], [HM2], [S1] and [S2]. We

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