J. Math. Kyoto Univ. (JMKYAZ)
37-4 (1998) 567-584

Radiation condition for Dirac operators

By

Chris PLADDY, Yoshimi SAITŌ and Tomio UMEDA

1. Introduction

In the papers [6] and [7], results from the theory of pseudodifferential operators and spectral analysis of Schrödinger operators were combined to discuss the asymptotic properties of the Dirac operator

$$H=-i\sum_{j=1}^{3}\alpha_j\frac{\partial}{\partial x_j}+\beta+Q(x). \tag{1.1}$$

Here $i=\sqrt{-1}$, $x=(x_1, x_2, x_3)\in\mathbf{R}^3$ and α_j, β are the Dirac matrices, i.e., 4×4 Hermitian matrices satisfying the anticommutation relation

$$\alpha_j\alpha_k+\alpha_k\alpha_j=2\delta_{jk}I, \quad (j, k=1, 2, 3, 4) \tag{1.2}$$

with the convention $\alpha_4=\beta$, δ_{jk} being Kronecker's delta and I being the 4×4 identity matrix. The potential $Q(x) = (q_{jk}(x))$ is a 4×4 Hermitian matrix-valued function. In this paper we assume that $Q(x)$ is short-range in the sense that each element q_{jk} satisfies

$$\sup_{x\in\mathbf{R}^3}[(1+|x|)^{1+\varepsilon}|q_{jk}(x)|]<\infty \quad (x\in\mathbf{R}^3, j, k=1, 2, 3, 4), \tag{1.3}$$

where ε is a positive constant. The free Dirac operator H_0 is defined by

$$H_0=-i\sum_{j=1}^{3}\alpha_j\frac{\partial}{\partial x_j}+\beta. \tag{1.4}$$

The aim of this paper is to show how the Dirac operator and the Schrödinger operator are related to each other and how some properties of the Dirac operator and the solutions of the Dirac equation can be obtained from the corresponding properties of the Schrödinger operator. Since we have from the anticommutation relation (1.2)

$$(H_0)^2=(-\Delta+1)I, \tag{1.5}$$

we can anticipate a close relationship between these two operators. We also

Communicated by Prof. N. Iwasaki, January 25, 1994
Revised January 8, 1997
1991 Mathematics Subject Classification: 35J10, 35L50, 35P25, 35P99, 47A40