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On spherically symmetric stellar models in general relativity

By

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1. Introduction

Recently H. R. Beyer [1] and S. S. Lin [7] investigated the linearized equation for small perturbations near spherically symmetric equilibria of a self-gravitating gas in the Newtonian i. e., non-relativistic theory of stellar structure. For a long time astrophysicists had been believing that the associated differential equation reduces to a Strum-Liouville eigenvalue problem, already from the Eddington's work in 1919, without any mathematical proof. But this was not obvious, since the coefficients of the equation are quite singular. H. R. Beyer has closed this gap in [1], and independently S. S. Lin has in [7] in a wider context. The aim of this article is to carry their results to the corresponding problem in general relativity.

In order to do that, we must investigate the qualitative properties of the structure of solutions for the equation which governs the spherically symmetric static configurations of self-gravitating gas in general relativity, since the cefficients of the linearized equation for small perturbations are determined by these equilibrium configurations. The equation which governs spherically symmetric equilibria in general relativity is formulated by J. P. Oppenheimer and G. M. Volkoff in [11], 1939, and this has been familiar in astrophysical textbooks as the TOV equation. But still now no qualitative analysis of the structure of this equation can be found in mathematical literatures. Even in the non-relativistic theory, in which the equation for equilibria under the equation of state $p = Const.\rho^{\gamma}$ reduces to the wellknown Lane-Emden equation, no fully mathematically rigorous treatments were found untill in 1972/73 the work [6] by D. D. Joseph and T. S. Lundgren appeared, although many numerical computations and good intuitive analogies had been obtained as found in the Chandrasekhar's famous textbook [2]. Moreover when the relation $p = Const.\rho^{\gamma}$ holds only in asymptotic sense, for example in the equation of state for white dwarfs, there were no mathematical proof of the elementary fact that the radius of a solution obtained by the shooting methods is finite untill the work [8] by the author, 1984. In general relativity, the author did not know any mathematically rigorous study of the structure of the

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