K3 surfaces with order five automorphisms

By

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Introduction

Let $T$ be a normal projective algebraic surface over $\mathbb{C}$ with at worst quotient singular points (= Kawamata log terminal singular points in the sense of [Ka, Ko]). $T$ is called a log Enriques surface if the irregularity $h^1(T, \mathcal{O}_T) = 0$ and if a positive multiple $IK_T$ of the canonical Weil divisor $K_T$ is linearly equivalent to zero. Without loss of generality, we always assume from now on that a log Enriques surface has no Du Val singular points (see the comments after [Z1, Proposition 1.3]).

The smallest integer $l > 0$ satisfying $IK_T \sim 0$ is called the (global) index of $T$. It can be proved that $l \leq 66$ (cf. [Z1]). Recently, R. Blache [B1] has shown that $l \leq 21$. He also studied the “generalized” log Enriques surfaces where log canonical singular points are allowed.

Rational log Enriques surfaces $T$ can be regarded as degenerations of K3 or Enriques surfaces, which in turn played important roles in Enriques-Kodaira’s classification theory for surfaces. In [A], A. Alexeev [A] has proved the boundedness of families of these $T$. In 3-dimensional case, the base surfaces $W$ of elliptically fibred Calabi-Yau threefolds $\phi_{[D]} : X \rightarrow W$ with $D.c_2(X) = 0$ are rational log Enriques surfaces (cf. [O1–O4]).

Let $T$ be a log Enriques surface of index $l$. The Galois $\mathbb{Z}/l\mathbb{Z}$-cover

$$
\pi : Y := \text{Spec}_T \bigoplus_{i=0}^{l-1} \mathcal{O}_T(-iK_T) \rightarrow T
$$

is called the (global) canonical covering. Clearly, $Y$ is either an abelian surface or a K3 surface with at worst Du Val singular points. We note also that $\pi$ is unramified over the smooth part $T - \text{Sing } T$.

We say that $T$ is of Type $A_m$ or $D_n$ if $Y$ has a singular point of Dynkin type $A_m$ or $D_n$; $T$ is of actual Type $(\oplus A_m) \oplus (\oplus D_n) \oplus (\oplus E_k)$ if $\text{Sing } Y$ is of type $(\oplus A_m) \oplus (\oplus D_n) \oplus (\oplus E_k)$.

Around 1989, M. Reid and I. Naruki asked the second author about the uniqueness of rational log Enriques surface to Type $D_{19}$. The determinations of all isomorphism classes of rational log Enriques surfaces $T$ of Type $A_{19}$, $D_{19}$, $A_{18}$ and $D_{18}$ have been done in [OZ1, 2] (see also [R1]). As a corollary, the minimal

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