Illinois Journal of Mathematics
Volume 50, Number 1, Spring 2006, Pages vii–viii
S 0019-2082

JOSEPH LEO DOOB (1910–2004)

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Probability has been a subject of mathematical exploration for more than 300 years, but it was given a firm mathematical foundation much later by mathematicians such as Kolmogorov in Russia and Doob in America, beginning in the 1930s. Doob in his 1937 paper (but see Chapter 2 of his 1953 book *Stochastic Processes* for a more transparent treatment) gave the mathematical framework for the study of continuous parameter stochastic processes, that is, families of measurable functions indexed by a parameter such that the family is pointwise continuous in the parameter.

After the pioneering work of Lévy, Ville, and others, he began a wide-ranging development of martingale theory about 1940, the year of publication of his paper dealing with *processes of type E.* As David Blackwell observed, Doob's work did not catch on until he started to use the more exciting word *martingale* which had already been used much earlier by Bachelier in a more limited mathematical context. Martingale theory is the focus of one of the chapters, nearly 100 pages long, in his book *Stochastic Processes.* This treatise of over 650 pages has been one of the most important and influential books on probability since Laplace's 1812 book. Martingale theory plays an essential role in the study of Markov processes, mathematical statistics, information theory, financial mathematics, combinatorics, and many other parts of mathematics, science, and technology. The noncommutative version of martingale theory plays a significant role in mathematical physics.

In 1954, he showed how various classical potential theory concepts, such as the properties of the Perron-Wiener-Brelot solution of the first boundary value problem for harmonic functions on an arbitrary open set of an Euclidean space and arbitrary assigned boundary function correspond to properties of superharmonic functions on Brownian motion paths. He obtained similar results for the heat equation in 1955.

During the early part of his career, he made important contributions to complex function theory, ergodic theory, Markov process theory, martingale theory, boundary theory, and much else. He and his work became enormously influential. He retired from teaching at the age of 68. Although he claimed that he was also retiring from mathematics as well, he continued to be mathematically active. This included writing a number of papers and two books;