GROUP EXTENSIONS AND TWISTED COHOMOLOGY THEORIES

BY

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Introduction

In this paper we continue the study of group extensions initiated in [7]. The specific problem discussed there was the computation of extensions in the exact sequence of groups obtained by mapping a space into a principal fibration sequence. Here we consider the same problem, but in a different category—the category of spaces "over and under" a fixed space (see [9], [1]). This means in particular that the solution to the extension problem is given in terms of "twisted" cohomology operations [9], whereas in [7] only ordinary cohomology operations were needed.

In §1 we discuss the category we will use. In §2 we state our extension problem, and in §§3–4 we give a general solution. Finally, in §§5–6 we give applications of our theory—in §5 we compute the (affine) group of immersions of an n manifold in R^{2n-1}, while in §6 we compute the (affine) group of vector 1-fields on a manifold.

1. The Category $\mathfrak{X}_B$

Let B be a fixed topological space. We define a category $\mathfrak{X}_B$ as follows: an object of $\mathfrak{X}_B$ is an ordered triple $(E, \check{e}, \hat{e})$ such that E is a topological space, $\hat{e} : E \to B$ is a continuous function, and $\check{e} : B \to E$ is a section of $\hat{e}$, i.e., $\hat{e} \circ \check{e} = 1_B$. If $e = (E, \check{e}, \hat{e})$ and $y = (Y, \check{y}, \hat{y})$ are objects, we say that $g : e \to y$ is a map if $g : E \to Y$ is a topological map and if $\hat{y} \circ g = \hat{e}$ and $g \circ \check{e} = \check{y}$; see McClendon and Becker [9], [1]. We say that two maps in $\mathfrak{X}_B$ are homotopic if there exists a homotopy of $\mathfrak{X}_B$-maps connecting them. Thus, we have the concept of homotopy equivalence in $\mathfrak{X}_B$.

Let X be any space and $f : X \to B$ a map. If $e = (E, \check{e}, \hat{e})$ and $g : X \to E$ is a map such that $\hat{e} \circ g = f$, we say that g is an f-map. Two f-maps are f-homotopic if they are connected by a homotopy of f-maps.

Let $[X, f; e]$ be the set of f-homotopy classes of f-maps from X to E. If $A \subset X$ is a subspace, let $[X, A, f; e]$ be the set of rel A f-homotopy classes of f-maps $X \to E$ which send A to $\check{e}(B)$.

Let (K, k_0) be a pointed CW complex, and let $e = (F, \check{e}, \hat{e})$ be an object in $\mathfrak{X}_B$. We define $e^K = (E^K_B, \check{e}^K, \hat{e}^K)$ as follows: E^K_B is the space of all maps (with the compact-open topology) $g : K \to E$ such that $g(k_0) \in \check{e}(B)$ and $\hat{e} \circ g$ is constant. For all $b \in B$ and $k \in K$, $\check{e}^K(b)(k) = \check{e}(b)$; for all $g \in E^K_B$, $\hat{e}^K(g) = \hat{e} \circ g(k_0)$. Let $\Omega e = e^S$ and $Pe = e^I$, where $S = S^1$ and $I = [0, 1]$ with basepoint 0.

Received June 26, 1971

[1] Research supported by a grant from the National Science Foundation.