RELATIONS BETWEEN THE COVERING HOMOTOPY AND SLICING STRUCTURE PROPERTIES

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1. Introduction

Maps which have the slicing structure property (SSP) as defined in [6] are a very strong type of fiber map. Unlike Serre and Hurewicz fibrations they are not defined using a covering homotopy property. However it is known [8], [15] that Hurewicz fibrations over locally equiconnected spaces have the SSP. As a matter of fact in [15] West showed that a paracompact space B is locally equiconnected iff every Hurewicz fibration over B has the SSP.

The SSP was used in [2], [7] and [11] in connection with local homogeneity. In [7] it was used to determine homology and homotopy groups of spaces associated with a locally homogeneous space. In [2] and [11] it was used to show that locally homogeneous spaces are like manifolds.

In [1], [3] and [13] sufficient conditions for a map to be a Hurewicz fibration are given. In all three papers it was first proven that the map had the SSP and hence if the base is paracompact then the map is a Hurewicz fibration. The fact that Hurewicz fibrations over ANR's have the SSP was used by Raymond in [10] to show the local triviality of a map. In [9] Mostert used the SSP to study light maps and quotients of topological groups.

In all of the above, except [7], it should be noted that the SSP was used to obtain topological results, not algebraic ones. Maps with the SSP are topologically easier to work with than Hurewicz fibrations.

The purpose of this paper is to study the topological structure of maps with the SSP. In particular several sufficient conditions, depending on various types of covering homotopy property, are given for a map to have the SSP. Also those Hurewicz fibrations which have the SSP are characterized by the existence of a special lifting function. An interesting corollary of the above is (3.9) which gives a very weak condition for a map to be a singular fiber map.

2. Definitions and notation

In the remainder of this paper p will be a map from a space E to a space B. Conditions will be placed on E, p or B as needed. The term map is used to denote a continuous function.

All function spaces will have the compact open topology and a subbasic open set will be denoted by (C, U) where C is compact and U is open.

Received October 19, 1972.