EQUIVARIANT AND HYPEREQUIVARIANT COHOMOLOGY

BY

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0. Introduction

The notion of equivariant cohomology with supports and with coefficients in a sheaf (module bundle) is defined and studied in Section 1 (Section 2). Theorem 1.4 shows that, under certain conditions on the supports and on the coefficients, equivariant cohomology can be reduced to ordinary sheaf theoretic cohomology. In Section 3 this fact is used in the construction of an equivariant Ω-spectrum for equivariant cohomology when the coefficient module bundle and the family of supports are of a certain type (Theorem 3.2). In Section 4 hyperequivariant cohomology is introduced. Theorems 4.2 and 4.3 show that, under various assumptions (Remark 4.4), hyperequivariant cohomology can be reduced to equivariant cohomology and can be classified by a hyperequivariant Ω-spectrum. It should be noted that classically the notions of equivariant and hyperequivariant cohomology coincide due to the fact that a group of automorphisms of a space is also a group in the category of spaces. In this paper "equivariance" is based on *categorical* groups (in particular, group bundles) and "hyperequivariance" is based on *automorphism* groups (of equivariant systems).

1. Equivariant cohomology with sheaf coefficients

Let $\mathscr{A}$ be a sheaf of modules over a sheaf of rings $\mathscr{R}$ on a space X ($\mathscr{A}$ is an $\mathscr{R}$-module in the sense of [1, p. 4]). If $f\colon X \to Y$ is a continuous map and $\mathscr{A}'$ is an $\mathscr{R}'$-module on Y then any f-cohomomorphism of sheaves of modules

$$(k, r)\colon (\mathscr{A}', \mathscr{R}') \to (\mathscr{A}, \mathscr{R})$$

induces a map [1, p. 45] $k_Y\colon \Gamma(\mathscr{A}') \to \Gamma(\mathscr{A})$, the image of which has the structure of a $\Gamma(\mathscr{R}')$-module. Let γ be a compactly generated group bundle over a compactly generated space B [9, Section 1] and let $\xi \in C = (\text{Haus } CG \downarrow B)$ (see [6, pp. 46 and 181]) be a left γ-space for which $q\colon \xi \to \xi/\gamma$, the quotient map onto the space of orbits, is in C, i.e., ξ/γ is Hausdorff (in general, an object in C and the total space of that object will be denoted by the same letter). Let $\mathscr{A}$ be an $\mathscr{R}$-module on ξ. A *γ-structure* on $\mathscr{A}$, briefly denoted by $\mathscr{A}^k$, consists of an $\mathscr{R}'$-module $\mathscr{A}'$ on ξ/γ together with a q-cohomomorphism $(k, r)\colon (\mathscr{A}', \mathscr{R}') \to (\mathscr{A}, \mathscr{R})$. Define $\Gamma(\mathscr{A}^k)$, the $\Gamma(\mathscr{R}')$-module of γ-equivariant sections, by $\Gamma(\mathscr{A}^k) =$ image $k_{\xi/\gamma}$. If ϕ is a family of supports on ξ let

$$\Gamma_\phi(\mathscr{A}^k) = \Gamma(\mathscr{A}^k) \cap \Gamma_\phi(\mathscr{A}).$$

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