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HIGHER ORDER SWEEPING OUT

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1. Introduction

Let T act in a measure space X with measure m and $m(X) = 1$. We say T *sweeps out* [3] if $m(A) > 0$ implies $m(\bigcup_{n=1}^{\infty} T^{k_n}A) = 1$ for all increasing sequences (k_n). We will say T is *lightly mixing* if $m(A) > 0$ and $m(B) > 0$ imply

$$\liminf_{n \to \infty} m(T^n A \cap B) > 0.$$

Lightly mixing implies mildly mixing [5] and weakly mixing [2]. In particular, if T is lightly mixing, then T is mixing on a sequence of density one [3], [6]. It is shown in [1] that the conditions for sweeping out and lightly mixing are equivalent. The term sequence mixing is used in [1] but might be confused with mixing on a sequence.

The definitions of higher order sweeping out and higher order lightly mixing are given in §2. In §3 it is shown that k-sweeping out is equivalent to lightly k-mixing, $k \geq 1$. The examples of transformations that are partially k-mixing but not partially $(k+1)$-mixing [4] are also examples of transformations that are lightly k-mixing but not lightly $(k+1)$-mixing, $k \geq 1$. The construction in [1] for $k = 1$ is generalized in §3 to obtain transformations that are lightly k-mixing but not partially k-mixing, $k \geq 1$.

A transformation T *uniformly sweeps out* if for each set A of positive measure and $\varepsilon > 0$ there exists $N = N(A, \varepsilon)$ such that the measure of the union of any N iterates of A is greater than $1 - \varepsilon$. If T is mixing, then T uniformly sweeps out [3]. It is not known if the converse is true. Higher order uniform sweeping out is introduced and in §4 it is shown that $(2k-1)$-mixing implies uniform k-sweeping out, $k \geq 1$.

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