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BOUNDARY LOCALIZATION OF THE NORMAL FAMILY OF HOLOMORPHIC MAPPINGS AND REMARKS ON EXISTENCE OF BOUNDED HOLOMORPHIC FUNCTIONS ON COMPLEX MANIFOLDS

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1. Basic definitions and statements of results

A modern treatment of the normal family of holomorphic mappings between complex manifolds was given in [6]. We shall recall some of the basic definitions here.

Let M and N be two metric spaces. A subset F of $C(M, N)$, the set of continuous mappings between M and N, is called normal if every sequence of F contains a subsequence which is either relatively compact in $C(M, N)$ or compactly divergent. A sequence $\{f_i\} \subset C(M, N)$ is called compactly divergent if for any compact sets $K \subset M$ and $K' \subset N$ there exists n_0 such that $f_i(K) \cap K' = \emptyset$ for all $i \geq n_0$.

DEFINITION. A complex manifold N is said to be *taut* if for every complex manifold M, the set of all holomorphic mappings from M to N, denoted by $\mathrm{Hol}(M, N)$, is a normal family.

A subset $F \subseteq C(M, N)$ is called an equicontinuous family if for any $\varepsilon > 0$ and any point $x \in M$ there is a neighborhood U of x such that if $x' \in U$, then $d_N(f(x), f(x')) < \varepsilon$ for all $f \in F$. Here N is a metric space equipped with a metric d_N inducing its underlying topology.

DEFINITION. Let N be a complex manifold equipped with a metric d inducing its underlying topology. (N, d) is called a *tight* manifold if for every complex manifold M, $\mathrm{Hol}(M, N)$ is equicontinuous.

An equivalent theory to the normal family of holomorphic mappings concerning intrinsic measures on complex manifolds has been developed by Eisenman, Kobayashi, Royden [1], [3], [5] and others. We recall its definition here for future use.

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