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A NON-LINEAR METHOD FOR CONSTRUCTING CERTAIN BASIC SEQUENCES IN BANACH SPACES

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I. Introduction

A basic problem in Banach space theory is whether every infinite dimensional Banach space contains an isomorphic copy of c_0 or a subspace which is isomorphic to an infinite dimensional conjugate space. This is, of course, a first step toward the better known conjecture that every Banach space contains an isomorphic copy of c_0 or l_1 or an infinite dimensional reflexive subspace. In this paper, we exhibit a new technique for constructing infinite boundedly complete basic sequences and consequently separable conjugate subspaces. An interesting feature of this method is that it involves a non-linear approach, which is in sharp contrast with the linear nature of the problem we address. Our approach also combines techniques originating in the local theory of Banach spaces (Dvoretzky's theorem and concentration phenomenon) with infinite dimensional concepts like dentability and "transfinite slicing" of sets.

One application of this method is that Banach spaces with the *Analytic Radon-Nikodym Property* (ARNP) contain copies of infinite dimensional conjugate spaces. (Recall that a complex Banach space X is said to have the ARNP if every X-valued bounded analytic map on the open unit disc of the complex plane has radial limits almost surely). This class of spaces contains—besides those possessing the Radon-Nikodym property (RNP) (see [D-U])—all Banach lattices not containing c_0 [B-D] as well as all preduals of Von Neuman algebras [H-P]. What is needed for the proof is the following geometric characterization of such spaces established in [G-L-M]: *Every bounded subset of a Banach space with the ARNP has arbitrarily norm-small slices determined by Lipschitz and plurisubharmonic functions.* In the classical RNP setting (where the slices are determined by continuous linear functionals) the analogous statement (i.e. The existence of infinite dimensional conjugate spaces) was established in [G-M1]. Also shown there is the case where the "slices" are determined by a finite number of linear functionals i.e.

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