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A SIMULTANEOUS ALMOST EVERYWHERE CENTRAL LIMIT THEOREM FOR DIFFUSIONS AND ITS APPLICATION TO PATH ENERGY AND EIGENVALUES OF THE LAPLACIAN

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1. Introduction and summary

Let M be a compact C^∞ manifold of dimension d. A C^∞ metric $\mathscr{g}$ on M gives rise to the notions of α-potential and α-energy, $\alpha > 0$. These were discussed in [2] and [4] in the context of the asymptotic behaviour of the diffusion on M with generator $L = \frac{1}{2}\Delta + V$, where Δ is the Laplace operator associated with $\mathscr{g}$, and V is a vector field. In this paper we shall continue that discussion, assuming for simplicity $V = 0$ and $d \geq 2$. In particular, we shall prove an almost everywhere central limit theorem (a.e. CLT) for the occupation measures of the diffusion, a theorem similar to the one we proved in [5] for IID random variables. The occupation measures assume their values in a nuclear space. We shall exploit "exponential mixing" of the diffusion. As an application of our a.e. CLT, we shall recover the spectrum of the operator Δ from the development of the α-energy on a typical diffusion path. A classical CLT for the occupation measures can be found in [2].

For background material on α-potentials and α-energy in $\mathbf{R}^d$ we refer the reader to [8].

In the case of a compact Riemannian manifold, the α-potential kernels $\{g_\alpha, \alpha > 0\}$ were defined in [3] in terms of the fundamental solution of the heat equation. To be precise, if p is the solution of

$$\frac{\partial p}{\partial t}(t, x, y) = \tfrac{1}{2}\Delta_y p(t, x, y), \quad p(0^+, x, y) = \gamma \delta_x(y),$$

with γ the total Riemann measure of $(M, \mathscr{g})$, for $\alpha > 0$ we let

$$(1.1) \qquad g_\alpha(x, y) = \frac{1}{\Gamma(\alpha)} \int_0^\infty t^{\alpha-1}\{p(t, x, y) - 1\}\, dt, \quad x, y \in M.$$

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