A NEW CHARACTERIZATION OF DIRICHLET TYPE SPACES AND APPLICATIONS

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1. Introduction

Let $D$ be the unit disk of the complex plane $\mathbb{C}$ and $dA(z) = 1/\pi \, dx \, dy$ be the normalized Lebesgue measure on $D$. For $\alpha < 1$, let

$$dA_\alpha(z) = (2 - 2\alpha)(1 - |z|^2)^{1-2\alpha} \, dA(z).$$

The Sobolev space $L^{2,\alpha}$ is the Hilbert space of functions $u : D \to \mathbb{C}$, for which the norm

$$\|u\| = \left( \int_D |u| dA_\alpha(z) \right)^{1/2} + \int_\Delta (|\partial u/\partial z|^2 + |\partial u/\partial \bar{z}|^2) \, dA_\alpha(z)$$

is finite. The space $D_\alpha$ is the subspace of all analytic functions in $L^{2,\alpha}$. This scale of spaces includes the Dirichlet type spaces ($\alpha > 0$), the Hardy space ($\alpha = 0$) and the Bergman spaces ($\alpha < 0$). (The Hardy and Bergman spaces are usually described differently, however see Lemma 3 of Section 3.) Let

$$\hat{D}_\alpha = \{ g \in D_\alpha : g(0) = 0 \}$$

and let

$$\hat{P} = \{ g \text{ is a polynomial on } D : g(0) = 0 \}.$$

Clearly $\hat{P}$ is dense in $\hat{D}_\alpha$. Let $P_\alpha$ denote the orthogonal projection from $L^{2,\alpha}$ onto $\hat{D}_\alpha$. For a function $f \in L^{2,\alpha}$ it is possible to define the (small) Hankel operator with symbol $f$, $h_f^{(\alpha)}$, on $\hat{P}$ by (see also [W1])

$$h_f^{(\alpha)} = P_\alpha(fg).$$

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