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A NON-ARCHIMEDEAN ANALOGUE OF THE KOBAYASHI SEMI-DISTANCE AND ITS NON-DEGENERACY ON ABELIAN VARIETIES

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One way to state the Schwarz-Pick Lemma is to say that holomorphic maps from the unit disc to itself are distance decreasing in the hyperbolic (Poincaré) metric. The Kobayashi semi-distance on a complex analytic space is an intrinsically defined semi-distance with the property that holomorphic mappings are distance decreasing in the Kobayashi semi-distance and such that the Kobayashi semi-distance on the unit disc is just the hyperbolic distance coming from the Poincaré metric.

If d denotes the Kobayashi semi-distance on a complex analytic space X, it is possible that $d(x, y) = 0$ for two distinct points $x \neq y$ in X. For instance, if $X = \mathbf{C}$ is the complex plane, then $d(x, y) = 0$ for every x and y in $\mathbf{C}$. Therefore, if X is any analytic space and f is a non-constant holomorphic map from $\mathbf{C}$ into X, then $d(x, y) = 0$ for any two points x, y in the image of f by the distance decreasing property of holomorphic maps. Brody's Theorem, [Br], in its weakest formulation says that in the case that X is compact, this is the only way the Kobayashi semi-distance can degenerate. Namely,

THEOREM (BRODY). *Let X be a compact, complex analytic space. Then there exist two distinct points $x \neq y$ in X such that the Kobayashi semi-distance $d(x, y) = 0$ if and only if there exists a non-constant holomorphic map from $\mathbf{C}$ into X.*

In this paper, I define a non-Archimedean analogue of the Kobayashi semi-distance, and, using Berkovich's, [Ber], theory of non-Archimedean analytic spaces, I show that this semi-distance does not degenerate on Abelian varieties. In [Ch1] (or see [Ch2]), I showed that every non-Archimedean map from the affine line $\mathbf{A}^1$ into an Abelian variety must in fact be constant. Therefore, I view the main result of this paper as the first step in answering:

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