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# CELL-LIKE MAPS AND ASPHERICAL COMPACTA

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A class of compacta on which cell-like maps cannot raise dimension was presented by Daverman [Da]. That class is expanded here, by establishing that all compact metric spaces contain compacta of codimension one on which cell-like maps do not raise dimension.

Classical results promise that cell-like maps defined on 1-dimensional compacta do not raise dimension. Dranishnikov [Dr] proved the existence of an infinite dimensional compactum whose integral cohomological dimension equals 3, from which it follows by the Edwards-Walsh construction [Wa] that there is a cell-like map on a 3-dimensional compactum with infinite-dimensional image. More recently, Dydak and Walsh [DW] confirmed that the same phenomenon could occur with 2-dimensional domain. Daverman [Da] introduced a notion of strongly hereditarily aspherical compacta, showed that cell-like maps on such compacta do not raise dimension, and provided examples in dimensions up to 5 with this asphericity property. Davis and Januszkiewicz [DJ] presented methods which give higher dimensional examples, by providing detailed elaborations of Gromov's useful idea [Gr] of hyperbolizing simplexes and polyhedra. A fortuitous consequence of the Cartan-Hadamard Theorem, for our purposes, is the fact that hyperbolization leads to asphericalization. One of our key results, a broad existence theorem, stems from techniques intimately related to this hyperbolization procedure. It is the following SHA Subset Theorem 3.1: Every compact metric space $X$ contains a 0-dimensional $F_\sigma$-subset $F$ such that all compact subsets of $X\backslash F$ are strongly hereditarily aspherical. As a striking consequence, every finite-dimensional, compact metric space $X$ contains a 0-dimensional $F_\sigma$-subset $F$ such that, for any cell-like map $p\colon X \to Y$ with infinite dimensional image, $p(F)$ is infinite-dimensional.

Corresponding to notions of hereditarily aspherical and strongly hereditarily aspherical compacta set forth in [Da], we introduce notions of hereditarily aspherical and strongly hereditarily aspherical maps. An issue still unresolved is whether the two types of compacta are distinct. Adding evidence for the suspicion that they are, we point out why the two types of maps are distinct. Among the highlights of §2 is Corollary 2.4, promising that a cell-like mapping which is strongly hereditarily aspherical over its image cannot raise dimension; near the end of the paper, in Theorem 3.7, we

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