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INDEPENDENCE AND MAXIMAL SUBGROUPS

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Dedicated to O. H. Kegel on the occasion of his 60th birthday

1. Introduction

In this paper G denotes a finite group and $M(G)$ the set of all maximal subgroups of G.

Recall that a matroid $(M, \mathcal{I})$ is a finite set M together with a set $\mathcal{I}$ of subsets of M (we call $X \subseteq M$ independent if and only if $X \in \mathcal{I}$) such that:

every subset of an independent set is independent, and every one-element subset is independent (i.e. $(M, \mathcal{I})$ is a simplicial complex)

and

if $A, B \in \mathcal{I}$ and $|A| < |B|$, then there is an $x \in B \setminus A$ such that $A \cup \{x\}$ is independent.

Examples of matroids are:

1. Let M be the (non-trivial) vectors of a finite vectorspace, $\mathcal{I}$ the linear independent sets.
2. Let M be the set of edges of a graph Γ and $\mathcal{I}$ the set of all circuit-free subsets of M.
3. Let $M = M_1 \cup M_2 \cup \cdots \cup M_l$ be a partition of M and

$$\mathcal{I} := \{X \subseteq M\colon\ |X \cap M_i| \leq 1 \text{ for all } i \leq l\}.$$

   Then $(M, \mathcal{I})$ is a matroid. This matroid is called the partition matroid of the partition $(M_i)_{i \leq l}$ of M.

Let $\mathcal{H} := (H_0 > H_1 > \cdots > H_l)$ denote a chief-series of G (i.e., a maximal chain of normal subgroups of G). Then $M(G)$ is the disjoint union of the sets $\mathcal{K}_i := \{U \in M(G)\colon\ H_i U = G,\ H_{i+1} \leq U\}$.

So, with $\mathcal{I}_{\mathcal{H}} := \{X \subseteq M(G)\colon\ |X \cap \mathcal{K}_i| \leq 1 \text{ for all } i < l\}$, we have a partition matroid $(M(G), \mathcal{I}_{\mathcal{H}})$. We call the independent subsets (i.e., the elements of $\mathcal{I}_{\mathcal{H}}$) $\mathcal{H}$-independent.

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