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TOEPLITZ ALGEBRAS ASSOCIATED TO ISOMETRIC FLOWS

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Introduction

Let M be a compact Riemannian manifold, and let $\Phi = \{\phi_t\}$ be a smooth one-parameter group of isometries of M. The group Φ is called an *isometric flow* on M. In this paper, we associate a Toeplitz C^*-algebra $\mathcal{T}(\Phi)$ to an isometric flow Φ on M, and begin to study how the C^*-algebraic properties of $\mathcal{T}(\Phi)$ are related to the geometric and topological properties of Φ.

The algebra $\mathcal{T}(\Phi)$ is defined as follows. Since each map ϕ_t is an isometry of M, it induces a unitary operator U_t on $L^2(M)$, where M is endowed with the usual measure coming from the Riemannian metric. Let D be the infinitesimal generator of the group $\{U_t\}$, and let P be the positive spectral projection of D. Then $\mathcal{T}(\Phi)$ is the C^*-subalgebra of $\mathcal{L}(L^2(M))$ generated by the set $\{PM_fP\colon f \in C(M)\}$, where M_f is pointwise multiplication by f.

Toeplitz C^*-algebras have been used by many researchers to study a variety of problems in geometry and analysis, and Toeplitz algebras have been particularly important in the study of flows. In addition, the C^*-algebras one obtains by this construction tend to be very interesting from an operator-algebraic point of view. Indeed, the Toeplitz algebra on the circle, i.e., the C^*-algebra generated by the unilateral shift, is precisely the Toeplitz algebra one gets by taking M to be the unit circle in the standard metric and where $\Phi = \{\phi_t\}$ is defined by letting ϕ_t be a rotation of $2\pi t$ for each t in $\mathbb{R}$.

There have been several papers written about Toeplitz operators and Toeplitz algebras associated to flows. Toeplitz algebras for irrational flows on tori were considered in [JX] and [JK]. These researchers examined the dependence of the Toeplitz algebra on the particular irrational flow chosen, and they also computed the K-theory of the Toeplitz algebras and their commutator ideals. It is natural to generalize the situation studied in [JX] and [JK] to topological flows on compact Hausdorff spaces, and this has been done in a series of papers. In [CMX], the spectral theory and index theory of Toeplitz operators were explored. In [MPX], the authors studied the Toeplitz algebra associated to a strictly ergodic flow, and they computed the K-theory of the Toeplitz algebra and some related algebras. In [MX], Toeplitz algebras for $\mathbb{R}^n$-actions were defined, and the results obtained specialize in the case $n = 1$ to give improved results

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