SOME UNIQUENESS THEOREMS ON RIEMANNIAN MANIFOLDS
WITH BOUNDARY

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1. Introduction

Let $M_n$ be a differentiable manifold of dimension $n$, and $X: M_n \rightarrow E_{n+m}$ a mapping of $M_n$ into a Euclidean space $E_{n+m}$ of dimension $n + m$ for any $m > 0$. $M_n$, or rather $M_n$ together with the mapping $X$, is called an immersed submanifold of $E_{n+m}$ if the functional matrix of $X$ is of rank $n$ everywhere. The submanifold $M_n$ is said to be imbedded, if $X$ is one-one, that is, if $X(P) = X(Q)$, $P, Q \in M_n$, implies that $P = Q$. In particular, when $m = 1$, an immersed (imbedded) submanifold $M_n$ of the space $E_{n+m}$ is called an immersed (imbedded) hypersurface. Throughout this paper all manifolds are supposed to be of class $C^3$, and the dimension of a manifold $M_n$ is understood to be $n$.

Now let us consider an oriented immersed manifold $M_n$. Then to each point $P \in M_n$ there is a unique linear space $N$ of dimension $m$ normal to $X(M_n)$ at the point $X(P)$. For any unit normal vector $e_r(P)$ at the point $X(P)$ in the space $N$, we put

\begin{align*}
I &= dX \cdot dX, \\
II_r &= de_r \cdot dX, \\
III_r &= de_r \cdot de_r,
\end{align*}

where $dX$ and $de_r$ are vector-valued linear differential forms on $M_n$, and the dot denotes the scalar product of two vectors in the space $E_{n+m}$. The eigenvalues $k_{r1}, \ldots, k_{rn}$ of $II_r$ relative to $I$ are called the principal curvatures of the manifold $M_n$ associated with the unit normal vector $e_r(P)$. If the Gauss-Kronecker curvature $K_r = k_{r1} \cdots k_{rn}$ associated with the vector $e_r(P)$ is nonzero, the reciprocals $1/k_{r1}, \ldots, 1/k_{rn}$, called the radii of principal curvatures associated with the vector $e_r(P)$, are the eigenvalues of $II_r$ relative to $III_r$, which is also positive definite due to the assumption $K_r \neq 0$. In this case we introduce the $\alpha$th elementary symmetric function

\begin{equation}
(\alpha) P_{\alpha} = \sum 1/k_{r1} \cdots 1/k_{r\alpha} \quad (1 \leq \alpha \leq n).
\end{equation}

If $M_n$ is a hypersurface, then at each point $X(P)$ of $M_n$ there is only one unit normal vector $e_r$, and for $P_{\alpha}$ associated with it we shall simply write $P_{\alpha}$.

Let $M_n$ be a closed oriented Riemannian manifold immersed in a Euclidean space $E_{n+m}$. By a normal frame $xe_{n+1} \cdots e_{n+m}$ on the manifold $M_n$ we mean a point $X$ of the manifold $M_n$ and an ordered set of mutually perpendicular unit vectors $e_{n+1}, \ldots, e_{n+m}$ normal to the manifold $M_n$ at the point $X$. This research was partially supported by the United States Air Force Office of Scientific Research of the Air Research and Development Command.

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