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ON THE DECOMPOSITION MATRICES OF THE QUANTIZED SCHUR ALGEBRA

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0. Introduction and general notation

0.1. The aim of this paper is to give a proof of the decomposition conjecture for the quantized Schur algebra [LeTh, Conjecture 5.2], which generalizes the theorem of Ariki (see [A]) on the decomposition numbers of the Hecke algebra of type A. More precisely, let $\bigwedge^\infty$ be the level-1 Fock space of type A, and let $\mathbf{B}^\pm$ be the bases of $\bigwedge^\infty$ introduced in [LeTh]. The decomposition conjecture links the decomposition matrices of the quantized Schur algebra and the basis $\mathbf{B}^+$. Our proof consists of two steps. First we express $\mathbf{B}^\pm$ in terms of some Kazhdan-Lusztig polynomials. Second, we note that a simple module of the quantized Schur algebra can be pulled back to a simple module of the Lusztig integral form of the quantized enveloping algebra of $\mathfrak{sl}_k$ (denoted by $\mathbf{U}(\mathfrak{sl}_k)$). Thus, the Lusztig conjecture for the dimension of the simple $\mathbf{U}(\mathfrak{sl}_k)$-modules at roots of unity identifies the entries of the decomposition matrices with some Kazhdan-Lusztig polynomials. It suffices to observe that these

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