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CORRECTION TO "THE COHOMOLOGY OF A COXETER GROUP WITH GROUP RING COEFFICIENTS"

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In Remark 5.8 on pages 309–310 of [1], I wrote, "It follows easily from Theorem A that if $L(W, S)$ is a Cohen-Macaulay complex of dimension $(n-1)$, then W is a virtual duality group of dimension n." In fact, as I explain below, a further hypothesis is required.

Suppose that L is an m-dimensional simplicial complex, that T is (the vertex set of) a simplex in L, and that $|T|$ denotes the number of vertices in T. A neighborhood of the barycenter of T in L is homeomorphic to the cone on $S^{|T|-1}Lk(T, L)$, the $(|T|-1)$-fold suspension of the link of T. Let $c_T : L \to S^{|T|}Lk(T, L)$ be the map that is the identity on this neighborhood and collapses its complement to a point. Consider the following condition:

(d) for each simplex T in L, $c_T^* : H^*(S^{|T|}Lk(T, L)) \to H^*(L)$ is injective.

Not every Cohen-Macaulay complex L satisfies condition (d). For example, if L is a triangulation of an m-disk, then condition (d) fails when T is a vertex in its interior.

The correct statement for Remark 5.8 is that if $L(= L(W, S))$ is an $(n-1)$-dimensional Cohen-Macaulay complex for which condition (d) holds, then W is a virtual duality group of dimension n. To see this, first note that $H^*(K^S) \cong H^*(L)$ and that, by excision, $H^*(K^S, K^{S-T}) \cong \overline{H}^*(S^{|T|}Lk(T, L))$ for any simplex T in L (i.e., for any $T \in \mathscr{S}^f_{>\varnothing}$). So, condition (d) is equivalent to the condition where $H^*(K^S, K^{S-T}) \to \overline{H}^*(K^S)$ is injective. Thus, if L is Cohen-Macaulay and condition (d) holds, then $\overline{H}^*(K^{S-T})$ is concentrated in dimension $(n-1)$ for any $T \in \mathscr{S}^f$. It then follows from Theorem A that $H_c^*(\Sigma)$ is concentrated in dimension n, and hence, W is a virtual duality group.

References

[1] M. W. Davis, *The cohomology of a Coxeter group with group ring coefficients*, Duke Math. J. **91** (1998), 297–314.

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