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TRILINEAR RESONANT INTERACTIONS OF SEMILINEAR HYPERBOLIC WAVES

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1. Introduction. An important question in the study of nonlinear equations is the understanding of the asymptotic behavior of families of solutions or of approximate solutions. One of the difficulties is the nonlinear interaction of oscillations. This phenomenon has been studied for special classes of solutions such as oscillatory functions with phases and amplitudes. This is the domain of geometric optics. In some cases, general families of bounded solutions have also been considered. The present paper enters the latter category, but is highly inspired by the results of nonlinear geometric optics.

Geometric optics provides a precise description of the asymptotic structure of solutions to both linear (e.g., [Wh]) and nonlinear (see [J], [HK], [MR], [HMR], [JMR1], [JMR2], [S], ...) hyperbolic PDEs whose initial data $u^{\varepsilon}(0,x) = u_0(x) + \varepsilon^m u_1(x, \vec{\varphi}_0(x)/\varepsilon)$ have rapid oscillations with prescribed phases $\vec{\varphi}_0$ and a single prescribed scale $1/\varepsilon$. Such solutions are important in both theory and applications (see [L], [KK]). In the lowest approximation, oscillations with different spatial scales interact only when $m = 0$, since otherwise the amplitudes of higher-frequency oscillations become negligible relative to those of lower frequencies. Taking $m = 0$ requires L^∞ estimates. Systems that are wellposed in L^∞ occur mostly in one spatial dimension. For such systems, arbitrary uniformly bounded sequences of initial data may be considered, but comparatively little is known about the asymptotic structure of the resulting solutions. Since the oscillations in such solutions may be extremely complex, a natural problem is to determine their nonoscillatory part, which may be identified with the weak limit of the sequence of solutions. The highly oscillatory nature of the solutions manifests itself in the absence of strong convergence in general. Of course, some information about those oscillations is generally needed in order to determine the weak limit.

The asymptotic structure of uniformly bounded solutions depends strongly on the nature of the nonlinearity. For entropy solutions of a genuinely nonlinear conservation law (see [T1]) or of a pair of such equations in one spatial dimension (see [DP]), oscillations of amplitude $O(1)$ do not persist for positive times; so a sequence of solutions converges strongly to the solution whose initial data is the weak limit of the initial data. Although oscillations do persist in linear systems, the weak limit of a sequence of solutions to such systems is just the solu-

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