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INFINITE-DIMENSIONAL LINKING

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1. Introduction. The concept of linking plays a fundamental role in variational principles. The usual definition is as follows (cf., e.g., [BBF]).

Definition 1. Let B be a closed subset of a Banach space E, and let Q be a submanifold of E with relative boundary ∂Q. For $A \subset E$, let C_A denote the set of all $\varphi \in C(E, E)$ that leave points of A fixed. Then $A = \partial Q$ links B if

$$A \cap B = \phi \tag{1.1}$$

and

$$\varphi(\bar{Q}) \cap B \neq \phi, \quad \varphi \in C_A. \tag{1.2}$$

As an example of the importance of linking, we have the following theorem.

THEOREM 1.1. *Let G be a C^1 functional on E, and let $A, B \subset E$ satisfy*

$$a_0 := \sup_A G \leqslant b_0 := \inf_B G, \tag{1.3}$$

$$A \quad \textit{links} \quad B, \tag{1.4}$$

and

$$c := \inf_{\varphi \in C_A} \sup_{u \in Q} G(\varphi(u)) < \infty. \tag{1.5}$$

Then there is a sequence $\{u_k\} \subset E$ such that

$$G(u_k) \to c, \quad G'(u_k) \to 0. \tag{1.6}$$

If one adds a compactness condition such as the Palais-Smale hypothesis, then one obtains a solution of

$$G(u) = c, \quad G'(u) = 0. \tag{1.7}$$

An examination of Definition 1 shows that it has the following shortcomings.

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