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RANDOM MATRICES, VIRASORO ALGEBRAS, AND NONCOMMUTATIVE KP

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0. Introduction. *What is the connection of random matrices with integrable systems? Is this connection really useful?* The answers to these questions lead to a new and unifying approach to the theory of random matrices. Introducing an appropriate time t-dependence in the probability distribution of the matrix ensemble leads to vertex operator expressions for the n-point correlation functions (probabilities of n eigenvalues in infinitesimal intervals) and the corresponding Fredholm determinants (probabilities of no eigenvalue in a Borel subset E). The latter probability is a ratio of τ-functions for the KP equation, whose numerator satisfies partial differential equations (PDE). These PDEs *decouple* into the sum of two parts: a Virasoro-like part, depending on time only, and a Vect (S^1)-part depending on the boundary points A_i of E. Upon setting $t = 0$ and using the KP hierarchy to eliminate t-derivatives, these PDEs lead to a hierarchy of nonlinear PDEs, purely in terms of the A_i. These PDEs are nothing else but the KP hierarchy for which the t-partials, viewed as commuting operators, are replaced by noncommuting operators in the endpoints A_i of the E under consideration. When the boundary of E consists of one point, and also for

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