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A REGULARIZED SIEGEL-WEIL FORMULA ON $U(2,2)$ AND $U(3)$

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1. Introduction. In their enormous paper [13], S. Kudla and S. Rallis proved the *first term identity* for the dual reductive pair $Sp(n), O(m)$ for arbitrary n and m. This is a significant generalization of the classical Siegel-Weil formula [25] in the case of symplectic and orthogonal groups, which is valid under the convergent range $m > 2n+2$. The formula gives an identity between a *theta integral* and a value of an Eisenstein series. Outside the convergent range, the theta integral may not converge and the Eisenstein series may have a pole at the critical value. Kudla and Rallis [13] show us how to *regularize* the theta integral so that the resulting object becomes a meromorphic function. Then they establish a relation between the *first terms* in the Laurent expansions of the Eisenstein series and the regularized theta integral at their critical values. This explains the name "first term identity." In another paper in collaboration with D. Soudry [14], Kudla and Rallis also proved a *second term identity*, which involves a comparison between the next terms of the two Laurent series for $Sp(2)$ and $O(4)$.

The purpose of this paper is to establish the analogous first and second term identities for the dual reductive pair $U(2,2)$, $U(3)$ that falls outside the convergent range for the unitary group case. The methods of this paper follow those of [13] and [14]. We omit some of the proofs when they are parallel to the symplectic-orthogonal case and give a detailed discussion when the unitary group case requires a separate treatment.

The outline of this paper is as follows. In Section 2, we define the regularized theta integral $\mathscr{E}(g,s,\varphi)$ associated to $U(2,2)$, $U(3)$ with φ an element in the space $S(V(\mathbb{A})^2)$ of the *oscillator representation* of $U(2,2)\times U(3)$. We see that $\mathscr{E}(g,s,\varphi)$ has at most a double pole at $s=1$. We then record in Section 3 some information regarding the composition series, *exponents*, and *singularity* of the local components of the induced representations $I(1/2)$ and $I_{P_1}(1)$ required in the proof of the main results. In Section 4, we show that our Siegel Eisenstein series $E(g,s,\Phi)$ has at most a simple pole at $s=1/2$, and we prove the first main theorem of the paper. Explicitly, let $A_{-1}(g,\Phi)$ be the leading term in the Laurent expansion of $E(g,s,\Phi)$ at $s=1/2$, and let $B_{-2}(g,\varphi)$ be that of $\mathscr{E}(g,s,\varphi)$ at $s=1$, where Φ is the *section* associated to φ. Then we have the following theorem.

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