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DIFFERENTIABLE CR MAPPINGS AND CR ORBITS

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1. Introduction. The geometric behaviour of a CR map $F : M \to M'$ between CR manifolds has been the object of some interest in the last years (see, e.g., [DP]). Our main purpose is to give a complete description when F is one-to-one (Theorem 1). More generally, we study the images of CR orbits when F is supposed to be only locally proper or proper (Theorem 2). We also study the propagation of the rank of F along orbits in a more general setting (Proposition 3.1). We assume that M, M' are connected and locally embeddable in $\mathbb{C}^N$.

In [CR] we proved the following.

THEOREM A [CR, Th. 2]. *Let F be one-to-one and M, M' have the same CR dimension. Then F is a diffeomorphism at all minimal points of M. M, M', and F are supposed to be of class $C^{2,\alpha}$, $0 < \alpha < 1$.*

At nonminimal points, this theorem fails to hold, as in the well-known example of S. Bell; there F is given by $\mathbb{C} \times \mathbb{R} \ni (z,t) \overset{F}{\mapsto} (z,t^3) \in \mathbb{C} \times \mathbb{R}$.

Theorem 1 shows that this example is in fact the prototype of a differentiable CR homeomorphism and gives in some way a complete geometric description of this kind of map.

THEOREM 1. *Let F be one-to-one, and let M and M' have the same CR dimension. Then*

(i) *F sends diffeomorphically local and global orbits onto local and global orbits, respectively, and*

(ii) *the rank of F is constant along the orbits (local or global).*

Here again, M, M', and F are supposed to be of class $C^{2,\alpha}$.

The definitions of "minimal point" and "orbit" are in Section 2.

For real *hypersurfaces*, it is known that local and global orbits (in this case, complex hypersurfaces) are in one-to-one correspondence when F is even only *continuous*. This is proved in [DP].

Theorem 1 evidently entails the following improvement of Theorem A.

(0) *If M or M' is (globally) minimal, then F is a diffeomorphism.*

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