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COMPUTATION OF THE DIFFERENCE OF TOPOLOGY AT INFINITY FOR YAMABE-TYPE PROBLEMS ON ANNULI-DOMAINS, II

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1. Introduction and statements of the results. For $\varepsilon > 0$, let $A_\varepsilon = \{x \in \mathbb{R}^n / \varepsilon < |x| < 1/\varepsilon\}$, $n \geqslant 3$. We consider the nonlinear elliptic problem

$$(P_\varepsilon) \quad \begin{cases} -\Delta u = u^{(n+2)/(n-2)}, u > 0 & \text{on } A_\varepsilon, \\ u = 0 & \text{on } \partial A_\varepsilon. \end{cases}$$

The motivation for investigating (P_ε) comes from its resemblance to the Yamabe problem in differential geometry, which consists of finding $u > 0$ satisfying

$$-\Delta u = u^{(n+2)/(n-2)} - \frac{n-2}{4(n-1)} R(x)u \quad \text{on } M,$$

where M is a Riemannian manifold of dimension n without boundary and $R(x)$ is the scalar curvature (see, for example, [1], [6], [9]).

We define on $H_0^1(A_\varepsilon)$ the functional

$$J_\varepsilon(u) = \frac{1}{2} \int_{A_\varepsilon} |\nabla u|^2 - \frac{n-2}{2n} \int_{A_\varepsilon} |u|^{2n/(n-2)} \tag{1}$$

whose positive critical points are solutions of (P_ε).

The problem (P_ε) is delicate from a variational viewpoint because of the possible existence of critical points at infinity, which are orbits of J_ε along which J_ε remains bounded, the gradient goes to zero, and the orbits do not converge (see [2] and [3]). To find the solutions of (P_ε) by studying the difference of topology between the level sets of J_ε, it becomes essential to evaluate the topological contribution of the critical points at infinity. In the first part of this work [7], we computed the difference of topology at infinity in the particular case of double blow-up for thin annuli-domains. Our aim in this paper is to compute it for expanding annuli ($\varepsilon \to 0$).

To state the main results, we need some notation. We denote by G_ε Green's function of the Laplace operator defined by

$$\forall\, x \in A_\varepsilon \begin{cases} -\Delta G_\varepsilon(x, \,) = c_n \delta_x & \text{on } A_\varepsilon, \\ G_\varepsilon(x, .) = 0 & \text{on } \partial A_\varepsilon, \end{cases} \tag{2}$$

where δ_x is the Dirac mass at x and $c_n = (n-2)\operatorname{meas}(S^{n-1})$.

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