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SOLUTIONS OF SUPERLINEAR ELLIPTIC EQUATIONS AND THEIR MORSE INDICES, I

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1. Introduction and main results. In this article, we consider solutions of the nonlinear elliptic problems

$$\begin{cases} -\Delta u = (u^+)^p & \text{in } \mathbb{R}^N, N \geqslant 3, \\ u \leqslant 1, \end{cases} \tag{I}$$

$$\begin{cases} -\Delta u = (u^+)^p & \text{in } \mathbb{R}^N_+, \\ u = 0 & \text{on } \{x_N = 0\}, \\ u \leqslant 1 \end{cases} \tag{II}$$

with finite Morse index. Here $u^+ = \max(u, 0)$, $\mathbb{R}^N_+ = \{(x_1, x_2, \ldots, x_N) \in \mathbb{R}^N, x_N > 0\}$.

Definition. The Morse index of $u, i(u)$, is the maximum of dimension of a subspace of $C_0^\infty(\mathbb{R}^N)$, the C^∞-functions h of $\mathbb{R}^N$ with compact support, such that the quadratic form $\int_{\mathbb{R}^N} |\nabla h|^2 - p \int_{\mathbb{R}^N} (u^+)^{p-1} h^2$ is nonpositive on this subspace.

We recall that B. Gidas and J. Spruck prove that there exist no nonzero positive solutions to (I) or (II) without using the Morse index (see Theorem 1.3, page 886 in [3]). We note that A. Bahri and P. L. Lions [1] prove a Liouville theorem: There exists no nonzero solution with finite Morse index of

$$\begin{cases} -\Delta u = |u|^{p-1} u & \text{in } \mathbb{R}^N, \\ |u| \leqslant 1 \quad u \in C^2(\mathbb{R}^N) \end{cases}$$

or

$$\begin{cases} -\Delta u = |u|^{p-1} u & \text{in } \mathbb{R}^N_+, \\ u = 0 & \text{on } \{x_N = 0\}, \\ |u| \leqslant 1 \quad u \in C^2(\mathbb{R}^N_+) \cap C(\mathbb{R}^N_+), \end{cases}$$

where

$$1 < p < \frac{N+2}{N-2}, \qquad N \geqslant 3.$$

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