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ON SOME DECOMPOSITION PROPERTIES FOR FACTORS OF TYPE II_1

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Introduction. A standard method of investigation in the study of von Neumann algebras is the decomposition of a given algebra into simpler ones by using such techniques as disintegration, tensor products, and cross-products. Through such decomposition techniques, von Neumann algebras have been reduced (to a certain extent!) to the study of type II_1 factors. These factors can thus be considered as the building blocks of the theory, but their structure is still far from being understood, except for a few remarkable classes, such as the hyperfinite type II_1 factor.

Along these lines, one usually tries to further decompose a given type II_1 factor M "around" an abelian or, more generally, a hyperfinite von Neumann subalgebra $R_0 \subset M$, for instance, as a cross-product $M = R_0 \times G = \overline{\mathrm{sp}}\, R_0\{u_g\}_{g\in G}$, with $G \cong \{u_g\}_{g\in G}$ a group of unitary elements of M acting on R_0. (Here and throughout the paper, $\overline{\mathrm{sp}}\, Y$ denotes the closed linear span of the set Y in the Hilbert norm given by the trace of the ambient type II_1 factor.) As this is rarely feasible, it is quite natural to allow the group $\{u_g\}_{g\in G}$ to be an algebra (a rather common "operation" in quantum theories), that is, to decompose M as $\overline{\mathrm{sp}}\, R_0N_1$ for some subalgebra N_1. One would then like N_1 to have a simple structure, ideally to be abelian or more generally hyperfinite.

At first glance, this might seem to be too strong a requirement, as perhaps imposing M itself to be hyperfinite. However, in [Po5], Popa found a large class of nonhyperfinite type II_1 factors having a decomposition of the form $M = \overline{\mathrm{sp}}\, R_0R_1$, with R_0, R_1 hyperfinite, coming from the theory of subfactors with finite Jones index. Factors having such a decomposition are called *thin* factors (see [Po5]), as to suggest being close to their hyperfinite building blocks.

We undertake in this paper a more detailed study of thin factors and of factors having other similar decomposition properties into abelian or hyperfinite algebras. Thus we show that all factors M with the property Γ of Murray and von Neumann [MvN2] can be decomposed as $M = \overline{\mathrm{sp}}\, R_0eR_0$ for some hyperfinite subfactor $R_0 \subset M$ and a projection $e \in M$, and also as $\overline{\mathrm{sp}}\, R_0R_1$ with R_1 a unitary conjugate of R_0. In particular, factors with property Γ are thin. Then we prove that if M is the cross-product of a hyperfinite algebra R_0 by a properly

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