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THE HYPERBOLIC MODULI SPACE OF FLAT CONNECTIONS AND THE ISOMORPHISM OF SYMPLECTIC MULTIPLICITY SPACES

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1. Introduction. In this paper we investigate multiplicity spaces for compact simple Lie groups. It was recently observed by L. Jeffrey [10] that in the case when all three coadjoint orbits belong to a small vicinity of $0 \in \mathfrak{k}^*$, symplectic multiplicity space coincides with the moduli space of $\mathfrak{k}$-valued flat connections on the sphere with three holes. It is also known that this result does not hold true for sufficiently large coadjoint orbits.

Though we do not know how to extend the results of [10], we reformulate the problem for the *hyperbolic* moduli space of flat connections that is obtained as a symplectic quotient of the space of $\mathfrak{g}$-valued connections with additional condition $\overline{A(z)} = -A(\bar{z})$ over the action of the gauge group $\overline{g(z)} = g^{-1}(\bar{z})$. In this setting we establish the isomorphism of the symplectic multiplicity space and the hyperbolic moduli space for *arbitrary* coadjoint orbits. This isomorphism is described explicitly.

The construction of the map between multiplicity space and moduli space is based on the simple observation that the holomorphic connection of the type

$$A(z) = \sum_i \frac{X_i}{z - z_i}\, dz \tag{1}$$

is flat. Here X_i belong to given coadjoint orbits. This construction was suggested by N. Hitchin in [9].

It is worth mentioning that both of our main results (Sections 4.2 and 4.3) are inspired by the work of V. Fock and A. Rosly [5], [6]. As a nice corollary of their description of the moduli space of flat connections, we obtain an isomorphism of the hyperbolic moduli space and the Poisson-Lie multiplicity space corresponding to an arbitrary Poisson structure on the compact group K.

The paper is organized as follows. In Section 2, we define symplectic and Poisson-Lie multiplicity spaces. Section 3 includes a review of the Goldman and Fock-Rosly descriptions of the moduli space of flat connections. There we also define the hyperbolic moduli space for the sphere with three holes. The iso-

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