Vol. 93, No. 1 DUKE MATHEMATICAL JOURNAL © 1998

THE RELATIVE LOG POINCARÉ LEMMA AND RELATIVE LOG DE RHAM THEORY

FUMIHARU KATO

1. Introduction

1.1. de Rham-Hodge theory and log geometry. Let $f : X \to Y$ be a smooth morphism of complex manifolds, and let $\Omega^\bullet_{X/Y}$ be the relative de Rham complex. Then the famous classical Poincaré lemma asserts that the natural morphism

$$f^{-1}\mathcal{O}_Y \xrightarrow{\sim} \Omega^\bullet_{X/Y}$$

of complexes is a quasi-isomorphism. The Poincaré lemma implies, providing that f is proper, the comparison of the de Rham cohomology and the Betti cohomology (de Rham theorem):

$$\mathrm{R}f_*\mathbf{Z} \otimes_{\mathbf{Z}} \mathcal{O}_Y \cong \mathbf{R}f_*\Omega^\bullet_{X/Y}.$$

Needless to say, it is essential to assume f to be smooth; even for semistable degeneration this argument can no longer be literally applied.

On the other hand, Hodge theorists have been interested in the degenerating behavior of the variation of the mixed Hodge structure (VMHS). It has been noticed that differentials with logarithmic poles take an essential role in studying limiting Hodge structures. Over the past few decades, a considerable number of studies have been made on this subject. J. H. M. Steenbrink proved in his exploring paper [17] the famous result for semistable degeneration, saying that the hypercohomology of the relative logarithmic de Rham complex of the central fiber is isomorphic to the Betti cohomology of a general fiber, the group of nearby cycles. Recently, the log geometry in the sense of J.-M. Fontaine, L. Illusie, and K. Kato has thrown new light on the subject. The new machinery provided by log geometry, such as log topological space (real blow-up), strikingly enables us to discuss very elegantly integral structures of degenerate VMHS, generalized Riemann-Hilbert correspondence, and so on; recent progress has been made by C. Nakayama and K. Kato [9], S. Usui [19] and [20], T. Fujisawa [2], and T. Matsubara [11].

In this paper we prove a generalization of the classical relative Poincaré lemma, the *relative log Poincaré lemma*, for log smooth morphisms $f : X \to Y$ of fs log analytic spaces satisfying suitable conditions. Here we assume neither that the

Received 27 February 1997.