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AFFINE HECKE ALGEBRAS AND RAISING OPERATORS FOR MACDONALD POLYNOMIALS

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CONTENTS

Introduction. In this paper, we introduce certain raising operators and lowering operators for Macdonald polynomials (of type A_{n-1}) by means of the Dunkl operators due to I. Cherednik. The raising operators we discuss below are a natural q-analogue of the raising operators for Jack polynomials introduced by L. Lapointe and L. Vinet [LV1], [LV2]. As an application of our raising operators, we prove the integrality of double Kostka coefficients which had been conjectured by I. G. Macdonald [Ma1] (apart from the positivity conjecture). We also include some application to a double analogue of the multinomial coefficients.

Let $\mathbb{K} = \mathbb{Q}(q,t)$ be the field of rational functions in two indeterminates (q,t), and let $\mathbb{K}[x]^W$ be the algebra of symmetric polynomials in n variables $x = (x_1, \ldots, x_n)$ over $\mathbb{K}$, W being the symmetric group $\mathfrak{S}_n$ of degree n. The *Macdonald polynomials* $P_\lambda(x) = P_\lambda(x; q, t)$ (or *symmetric functions with two parameters*, in the terminology of Macdonald [Ma1]), are a family of symmetric polynomials parametrized by partitions, and they form a $\mathbb{K}$-basis of $\mathbb{K}[x]^W$. One way to characterize these polynomials is, among others, to consider the joint eigenfunctions in $\mathbb{K}[x]^W$ for the commuting family of q-difference operators

$$(1) \qquad D_x^{(r)} = t^{\binom{r}{2}} \sum_{\substack{I\subset[1,n] \\ |I|=r}} \prod_{\substack{i\in I \\ j\notin I}} \frac{tx_i - x_j}{x_i - x_j} \prod_{i\in I} T_{q,x_i} \quad (r = 0, 1, \ldots, n).$$

The Macdonald polynomial $P_\lambda(x)$ is characterized as the joint eigenfunction of $D_x^{(r)}$ $(r = 0, 1, \ldots, n)$ that has the leading term $m_\lambda(x)$ under the dominance order

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