

NONTRIVIAL SOLUTIONS OF ELLIPTIC EQUATIONS WITH SUPERCRITICAL EXPONENT IN CONTRACTIBLE DOMAINS

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0. Introduction. Let us consider the problem

$$P(\Omega) \begin{cases} \Delta u + |u|^{p-2}u = 0 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \\ u \neq 0 & \text{in } \Omega, \end{cases}$$

where Ω is a bounded domain of \mathbb{R}^n , with $n \geq 3$, and $p > 2^* = 2n/(n-2)$ is a fixed exponent (2^* is the critical exponent for the Sobolev embedding $H_0^{1,2}(\Omega) \hookrightarrow L^p(\Omega)$). Our aim is to analyse the effect of the domain shape on the existence and multiplicity of solutions.

When $2 < p < 2^*$, the existence of positive and sign changing (nodal) solutions of $P(\Omega)$ does not depend on the shape of Ω . On the contrary, when $p \geq 2^*$, the solvability of the problem is strictly related to the shape of the domain. In fact, no solution can exist if Ω is starshaped, as a consequence of the well-known Pohozaev identity (see [27]), while, if Ω is, for example, an annulus, it is easy to find infinitely many solutions.

For $p = 2^*$, a general result of Bahri and Coron (see [4]) guarantees the existence of positive solutions in domains Ω having nontrivial topology (i.e., certain homology groups of Ω are nontrivial). Subsequently, existence and multiplicity of positive solutions have been stated, even in some contractible domains (see [11], [12], [17], [18], and [22]).

Several results on problems with critical nonlinearity have been obtained in recent years by many authors. These results relate existence, nonexistence and multiplicity of positive and nodal solutions to the shape of the domain or also to the properties of some perturbations of the equation by lower-order terms (see [1]–[3], [5]–[10], [20], [24], and [28]–[32]).

On the contrary, in the supercritical case the problem is still widely open. In particular, a natural question (raised by Rabinowitz) is extending the result of Bahri-Coron [4] to the case $p > 2^*$, that is, to find suitable conditions on the topology of Ω , that guarantee existence of nontrivial solutions for $p > 2^*$. (Other problems with supercritical growth are considered in [13], [14], and [15].)

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