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ON THE TEMPERED SPECTRUM OF QUASI-SPLIT CLASSICAL GROUPS

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§1. Introduction. One of the most striking aspects of the Langlands program is the conjectural relation between harmonic analysis and number theory. The proof of a conjecture of Langlands given in [20] shows that determining the poles of certain (conjectural) Langlands L-functions is equivalent to determining the nondiscrete tempered spectrum of reductive p-adic groups. The theory of endoscopy [16] and twisted endoscopy [13], [14] has proved particularly useful in giving a context within which to explore these problems, at least for classical groups (see the introduction and Section 3 of [21]).

Previously the authors separately determined the nondiscrete tempered spectrum of classical groups supported in their Siegel parabolic subgroups (see [8] and [21]) or, equivalently, computed the symmetric square and the exterior square L-functions for $GL_n(F)$ [21], as well as the Asai L-functions for $GL_n(E)$, where E is a quadratic extension of F [8]. In fact, these L-functions were determined for an arbitrary irreducible admissible representation of $GL_n(F)$ or $GL_n(E)$, accordingly. Finally, in [22], the second author addressed the problem for arbitrary maximal parabolic subgroups of split even special orthogonal groups. In terms of L-functions, the work in [22] determines the Rankin-Selberg product L-functions attached to irreducible admissible representations of $GL_n(F) \times SO_{2m}(F)$.

The purpose of the present paper is twofold. First we generalize the work in [22] to symplectic and quasi-split special orthogonal groups and thus, using [9], eventually determine the nondiscrete tempered spectrum of these groups. The second purpose is to remove a gap that existed in the proof of Theorems 7.8 and 8.1 of [22]. The final results of the present paper, Theorem 4.8 and Corollary 4.9, while having the similar main (regular) term R_G, have a different and more complicated singular contribution than the singular terms given in Theorems 7.8 and 8.1 of [22]. However, in Proposition 5.2 of the present paper, we manage to relate the singular terms from the two different versions to each other (see Remark 4.11). The reader of [22] must therefore consider Theorem 4.8 and Corollary 4.9 of the present paper as correct versions of Theorems 7.8 and 8.1 of [22], and it is most efficient if Sections 4 and 5 of the present paper are sub-

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