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HIGHER REGULATORS, HILBERT MODULAR SURFACES, AND SPECIAL VALUES OF L-FUNCTIONS

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0. Introduction. In this article, we establish a connection between elements in the K-theory of a Kuga-Sato-variety over a Hilbert modular surface and special values of (Asai) L-functions as predicted by A. Beilinson's general conjecture. Before we explain our result in more detail, let us briefly recall the conjecture. Let M be a pure (Chow) motive with coefficients in $\bar{\mathbb{Q}}$ and let $L(s, H^i_{\text{ét}}(M))$ be its L-function. Let, for $n > i/2 + 1$,

$$r_{\mathscr{H}} : H^{i+1}_{\mathscr{M}}(M_{\mathbb{Z}}, \mathbb{Q}(n)) \to H^{i+1}_{\mathscr{H}}(M \otimes \mathbb{R}, \mathbb{R}(n))$$

be Beilinson's regulator into absolute Hodge cohomology. The weak form of Beilinson's conjecture may now be stated as follows (see [Be2, § 3] and [Ja1, § 4]): There exists a $\bar{\mathbb{Q}}$ subspace $\mathscr{K}(i,n) \subset H^{i+1}_{\mathscr{M}}(M_{\mathbb{Z}}, \mathbb{Q}(n))$ such that

$$r_{\mathscr{H}}(\det \mathscr{K}(i,n)) = L(n, H^i_{\text{ét}}(M)) \det \mathscr{D}\mathscr{R}(i,n)$$

where $\mathscr{D}\mathscr{R}(i,n)$ is P. Deligne's $\bar{\mathbb{Q}}$-structure. Not many cases of this conjecture are known. We mention Beilinson's work [Be1, § 7] on the case of Dirichlet motives and for modular forms of weight 2 [Be2]. This paper also introduces the Eisenstein symbol, which is the main technical tool for all subsequent work. C. Deninger has treated the case of algebraic Hecke characters over imaginary quadratic fields [Den]. A. J. Scholl [Sch4] proved the conjecture in the case of modular forms of weight greater than 2. These are essentially all results outside the critical strip of the L-function (see [N] for a survey of the known results). Note that D. Ramakrishnan's work [Ram2] and [Ram3] on higher regulators on Hilbert modular surfaces concerns the left central point, which needs a different formulation of the conjecture due to the possible pole of the L-function at $s = 2$ coming from the existence of divisor classes modulo algebraic equivalence.

To explain our result, let us introduce some notation. Let F be a real quadratic field and let $G \subset \mathrm{Res}_{F/\mathbb{Q}}\mathrm{GL}_{2,F}$ be the subgroup of the Weil restriction of GL_2 as introduced in [Ra1, see (1.1.1)]. This group gives rise to a Shimura variety $S(\mathbb{C})$ with $G(\mathbb{A}_f)$ action. Consider the local system defined by the G-representation $V^{p,q} := \mathrm{Sym}^p V_2^\vee \otimes \mathrm{Sym}^q V_2^\vee$, where $V_2^\vee$ is the dual of the standard representation of GL_2. According to [BL, 1.3.9], the intersection cohomology with respect to the Baily-Borel compactification decomposes into

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