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ASYMPTOTIC OF THE DENSITY OF STATES FOR THE SCHRÖDINGER OPERATOR WITH PERIODIC ELECTRIC POTENTIAL

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1. Introduction. We are interested in the study of the spectral properties of the Schrödinger operator on $L^2(\mathbb{R}^n)$, with $n \geqslant 2$,

$$P_V = \sum_{j=1}^{n} D_{x_j}^2 + V(x), \tag{1.1}$$

where $D_{x_j} = -i\partial_{x_j} = (1/i)(\partial/\partial x_j)$, for $j = 1, \dots, n$.

The potential $x \mapsto V(x)$ is real and

$$V(\cdot) \in C^\infty(\mathbb{R}^n; \mathbb{R}). \tag{1.2}$$

Let Γ be a lattice on $\mathbb{R}^n$,

$$\Gamma = \left\{ \sum_{j=1}^{n} k_j e_j;\ k = (k_1, \dots, k_n) \in \mathbb{Z}^n \right\}, \tag{1.3}$$

where $\{e_1, \dots, e_n\}$ is a basis of $\mathbb{R}^n$.

We assume that the potential is periodic:

$$V(x + a) = V(x), \qquad \forall a \in \Gamma. \tag{1.4}$$

It is well known that P_V is essentially selfadjoint on $L^2(\mathbb{R}^n)$, starting from $C_0^\infty(\mathbb{R}^n)$, the space of C^∞ functions with compact support. For any function $f \in C_0^\infty(\mathbb{R})$, the bounded operator on $L^2(\mathbb{R}^n)$ $f(P_V)$ has, due to the ellipticity of P_V, a C^∞ kernel $K_{f(P_V)}(x, y)$. The Γ-periodicity of V shows that this kernel satisfies

$$K_{f(P_V)}(x + a, y + a) = K_{f(P_V)}(x, y), \qquad \forall a \in \Gamma, \quad \forall x, y \in \mathbb{R}^n, \tag{1.5}$$

and in particular the function $x \mapsto K_{f(P_V)}(x, x)$ is Γ-periodic:

$$K_{f(P_V)}(x + a, x + a) = K_{f(P_V)}(x, x), \qquad \forall a \in \Gamma, \quad \forall x \in \mathbb{R}^n. \tag{1.6}$$

(See [HS1] and [Sjö] for details.)

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