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A REPRODUCING KERNEL FOR NONSYMMETRIC MACDONALD POLYNOMIALS

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To Professor Reiji Takahashi on his 70th birthday

§0. Introduction. In this paper we propose a new formula of the Cauchy type for the nonsymmetric Macdonald polynomials of type A_{n-1}. This can be regarded as an explicit formula for the reproducing kernel of a certain scalar product on the polynomial ring of n variables. A similar result for nonsymmetric Jack polynomials was recently given by Sahi [S].

The nonsymmetric Macdonald polynomials $E_\lambda(x|q,t)$, introduced by Macdonald [Ma1], are characterized as the joint eigenfunctions in the polynomial ring of n variables $x = (x_1, \ldots, x_n)$, for the commuting family of q-Dunkl operators. (For the precise definition of $E_\lambda(x|q,t)$, see Section 1.) We define a meromorphic function $E(x; y|q,t)$ in $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ by

$$E(x;y|q,t) = \prod_{1\leqslant j<i\leqslant n} \frac{(qtx_iy_j;q)_\infty}{(qx_iy_j;q)_\infty} \prod_{1\leqslant i\leqslant n} \frac{(qtx_iy_i;q)_\infty}{(x_iy_i;q)_\infty} \prod_{1\leqslant i<j\leqslant n} \frac{(tx_iy_j;q)_\infty}{(x_iy_j;q)_\infty}. \tag{0.1}$$

The main result of this paper is the following.

THEOREM. *The function $E(x; y|q,t)$ has the following expansion in terms of nonsymmetric Macdonald polynomials:*

$$E(x;y|q,t) = \sum_{\lambda\in\mathbb{N}^n} a_\lambda(q,t)E_\lambda(x|q,t)E_\lambda(y|q^{-1},t^{-1}). \tag{0.2}$$

For each composition $\lambda \in \mathbb{N}^n$, the coefficient $a_\lambda(q,t)$ is given by

$$a_\lambda(q,t) = \prod_{s\in\lambda} \frac{1-q^{a(s)+1}t^{l(s)+1}}{1-q^{a(s)+1}t^{l(s)}}, \tag{0.3}$$

where, for each box $s \in \lambda$, $a(s)$ and $l(s)$ are the arm length and the generalized leg length of s in λ.

This theorem follows from Theorem 2.1 and Theorem 2.2.

After presenting preliminary discussion on nonsymmetric Macdonald polynomials, we formulate our main results in Section 2. In that section, we prove

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